Robot Motion Control and Planning

http://www.ceng.metu.edu.tr/~saranli/courses/ceng786

Lecture 5 – Roadmap Methods

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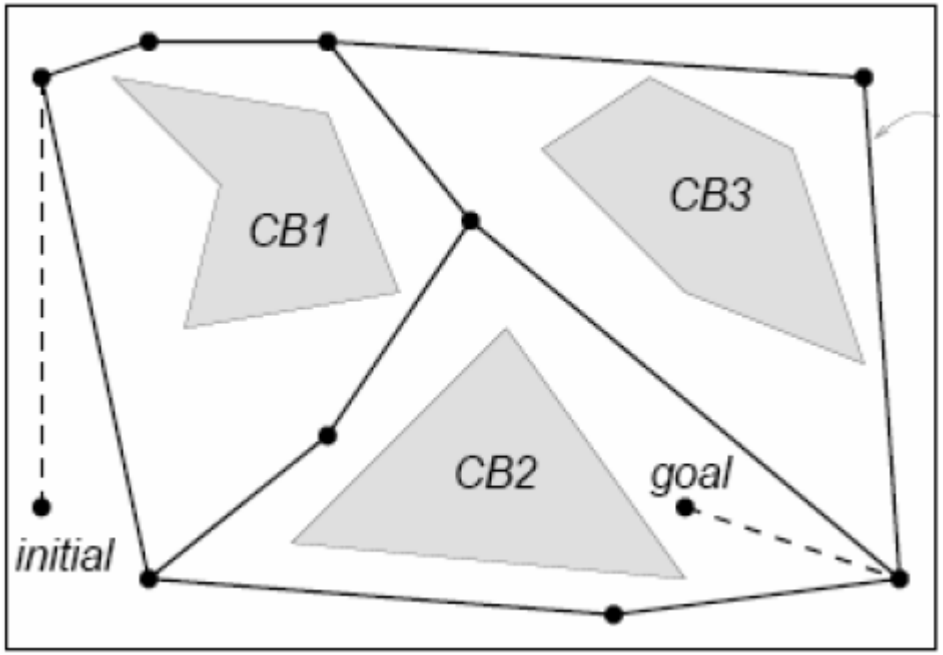
The Basic Idea

- Capture the connectivity of Q_{free} by a graph or network of paths.



RoadMap Definition

- A roadmap, RM, is a union of curves such that for all start and goal points in Q_{free} that can be connected by a path:
  - **Accessibility:** There is a path from q_{start} in Q_{free} to some q' in RM
  - **Departability:** There is a path from some q'' in RM to qgoal in Q_{free}
  - **Connectivity:** there exists a path in RM between q' and q''
  - **One dimensional**

RoadMap Path Planning

1. Build the roadmap
   - nodes are points in Q_{free} (or its boundary)
   - two nodes are connected by an edge if there is a free path between them

2. Connect start and goal points to the road map at point q' and q'', respectively

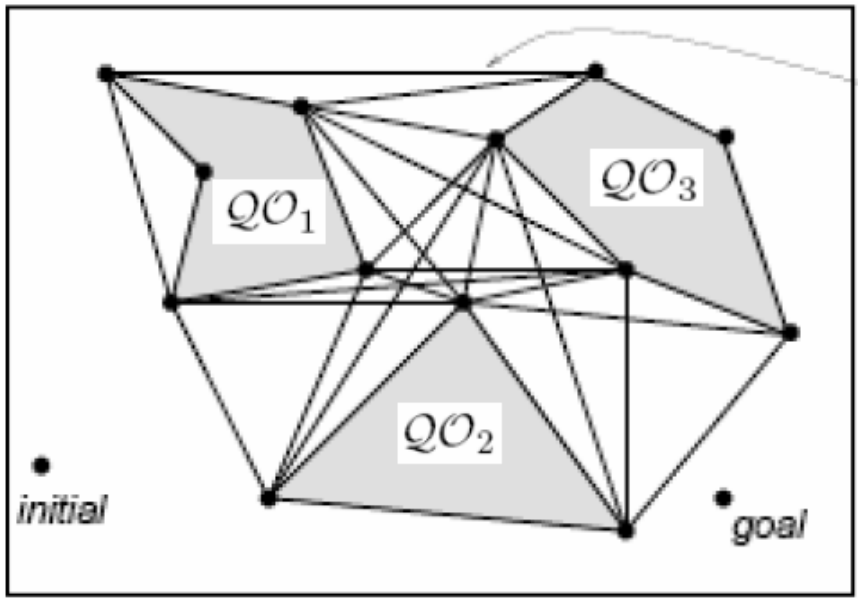
3. Find a path on the roadmap betwen q' and q''
   - The result is a path in Q_{free} from start to goal
   - Question: what is the hard part here?

Overview: Deterministic Methods

- Some need to represent Q_{free}, and some don't.
- are complete
- are complexity-limited to simple (e.g. low-dimensional) problems
  - example: Canny's Silhouette method (5.5)
    - applies to general problems
    - is singly exponential in dimension of the problem

Visibility Graph Methods

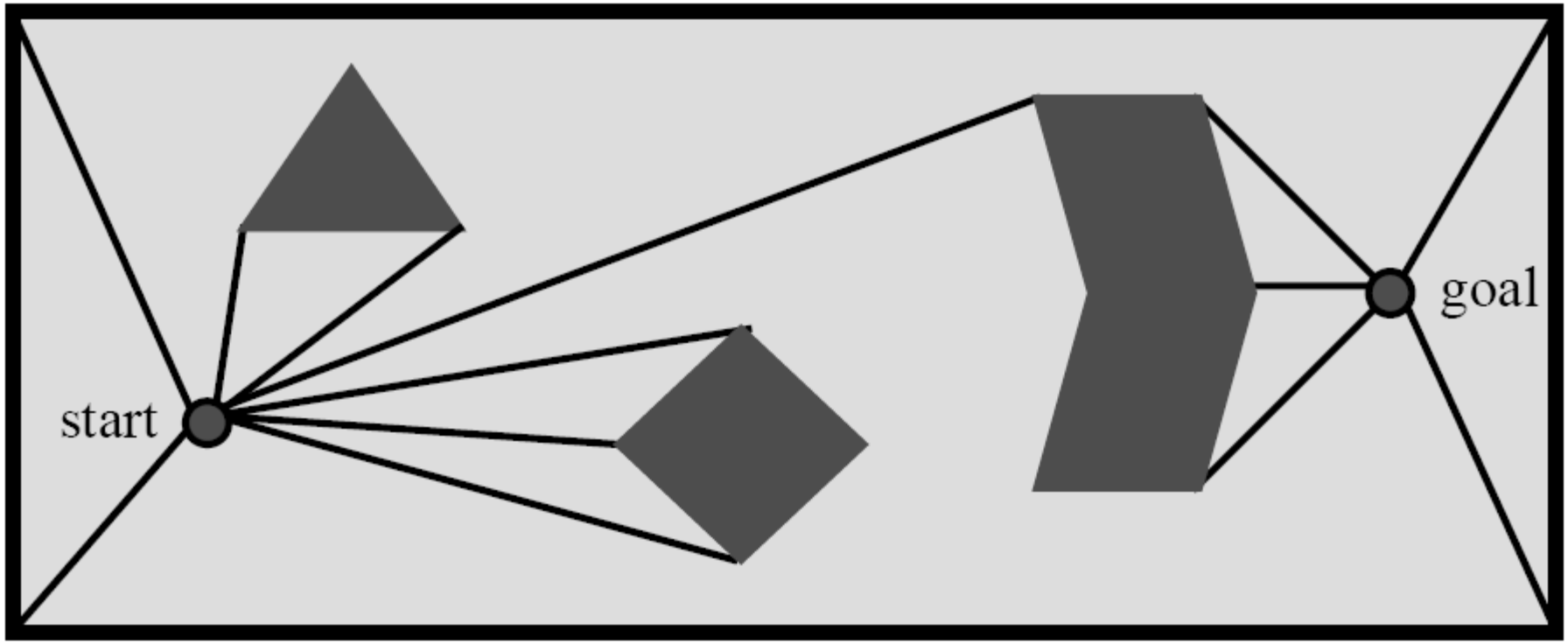
- Defined for polygonal obstacles
- Nodes correspond to vertices of obstacles
  - Nodes are connected if they are already connected by an edge on an obstacle
  - the line segment joining them is in free space
- Not only is there a path on this roadmap, but it is the *shortest* path
- If we include the start and goal nodes, they are automatically connected
- Algorithms for constructing them can be efficient
  - $O(n^3)$ brute force



The Visibility Graph in Action (Part 1)

- First, draw lines of sight from the start and goal to all “visible” vertices and corners of the world.

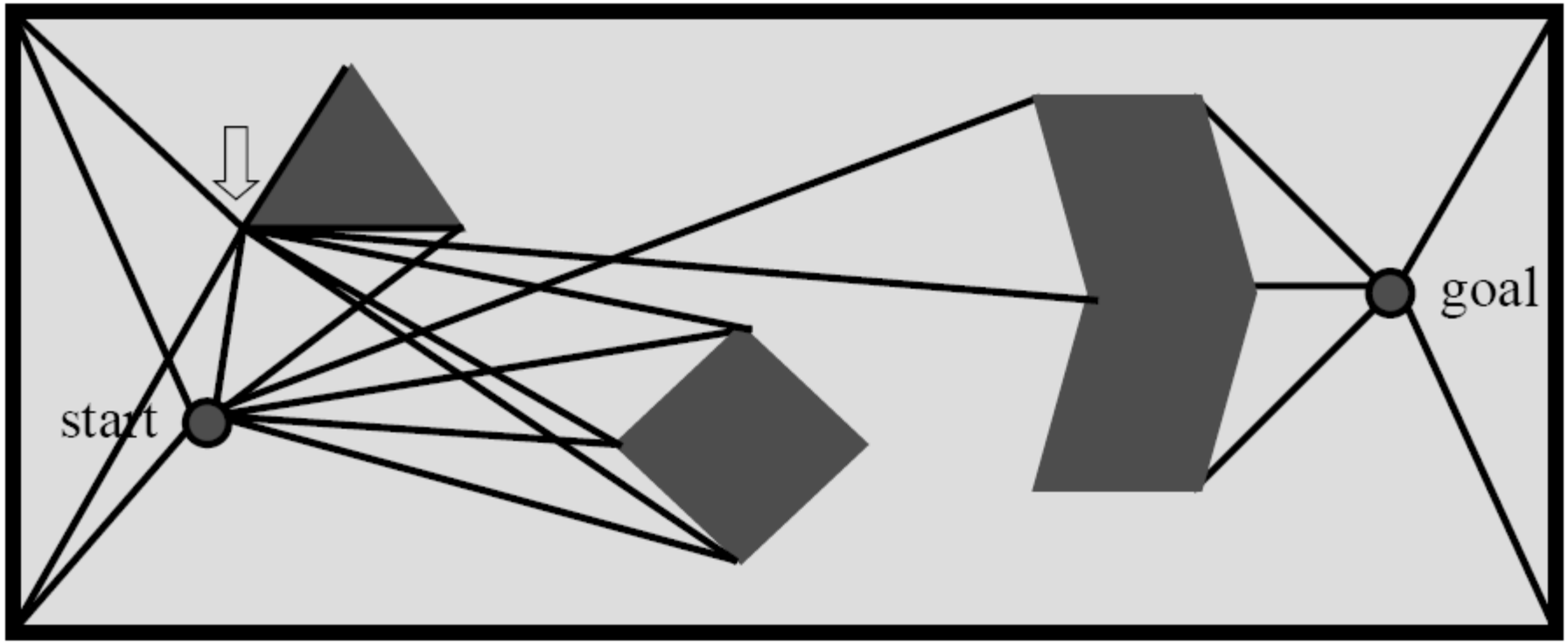
$$e_{ij} \neq \emptyset \iff sv_i + (1-s)v_j \in \mathrm{cl}(\mathcal{Q}_{\mathrm{free}}) \quad \forall s \in (0,1)$$



The Visibility Graph in Action (Part 2)

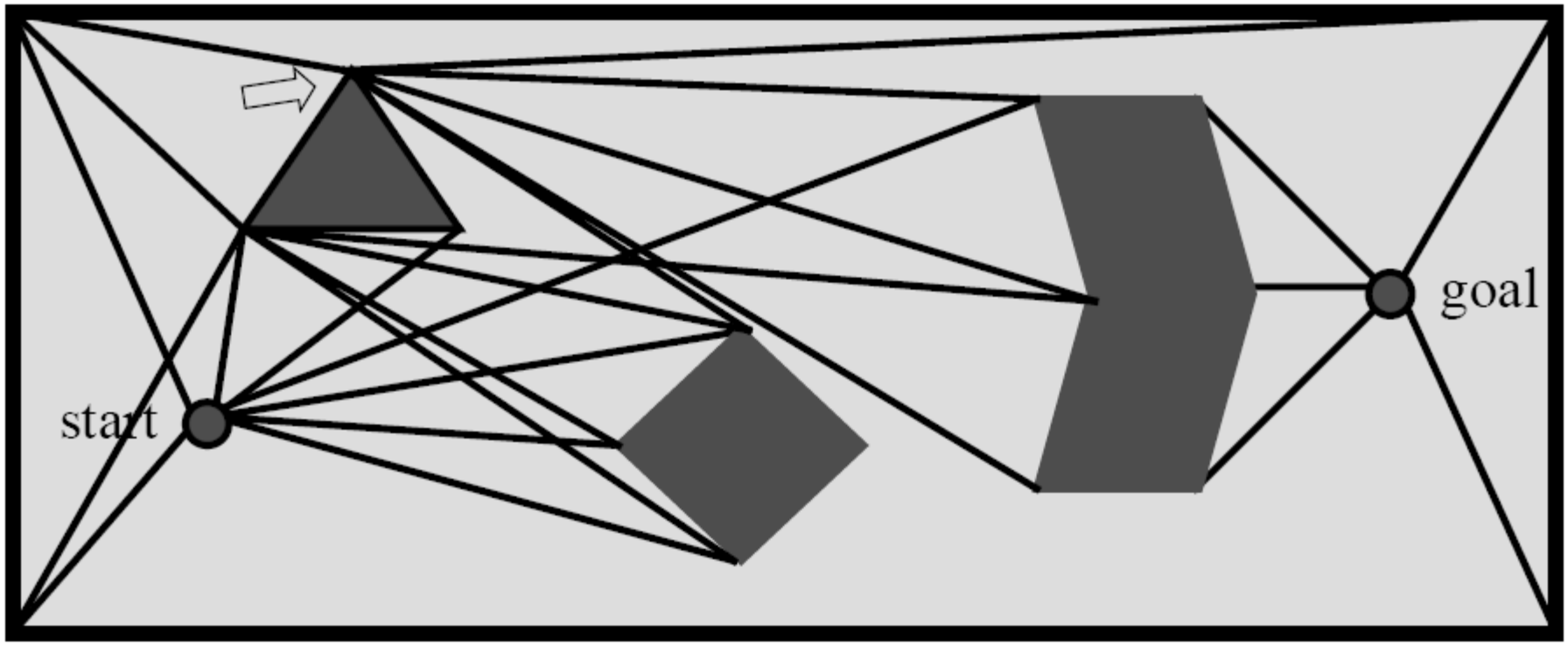
- Second, draw lines of sight from every vertex of every obstacle like before. Remember, lines along edges are also lines of sight.

$$e_{ij} \neq \emptyset \iff sv_i + (1-s)v_j \in \mathrm{cl}(\mathcal{Q}_{\mathrm{free}}) \quad \forall s \in (0,1)$$



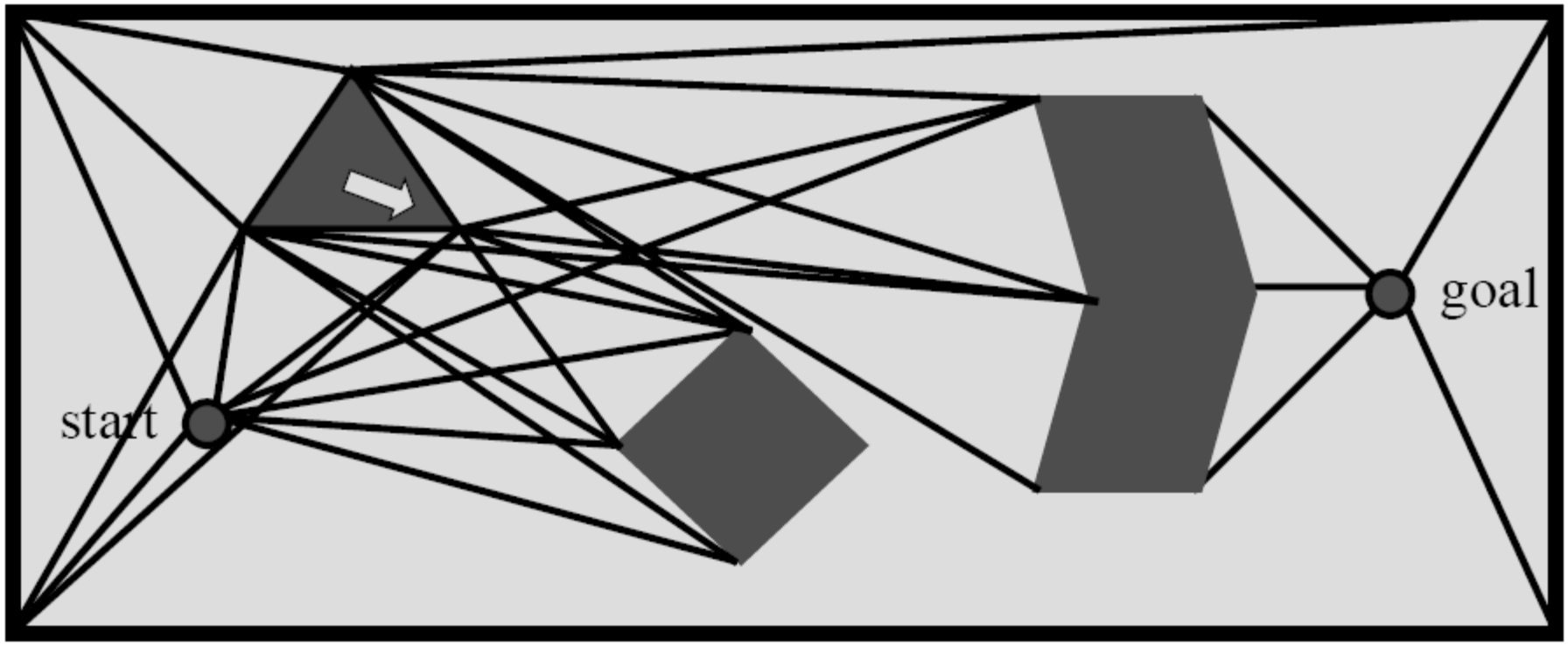
The Visibility Graph in Action (Part 3)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember, lines along edges are also lines of sight.



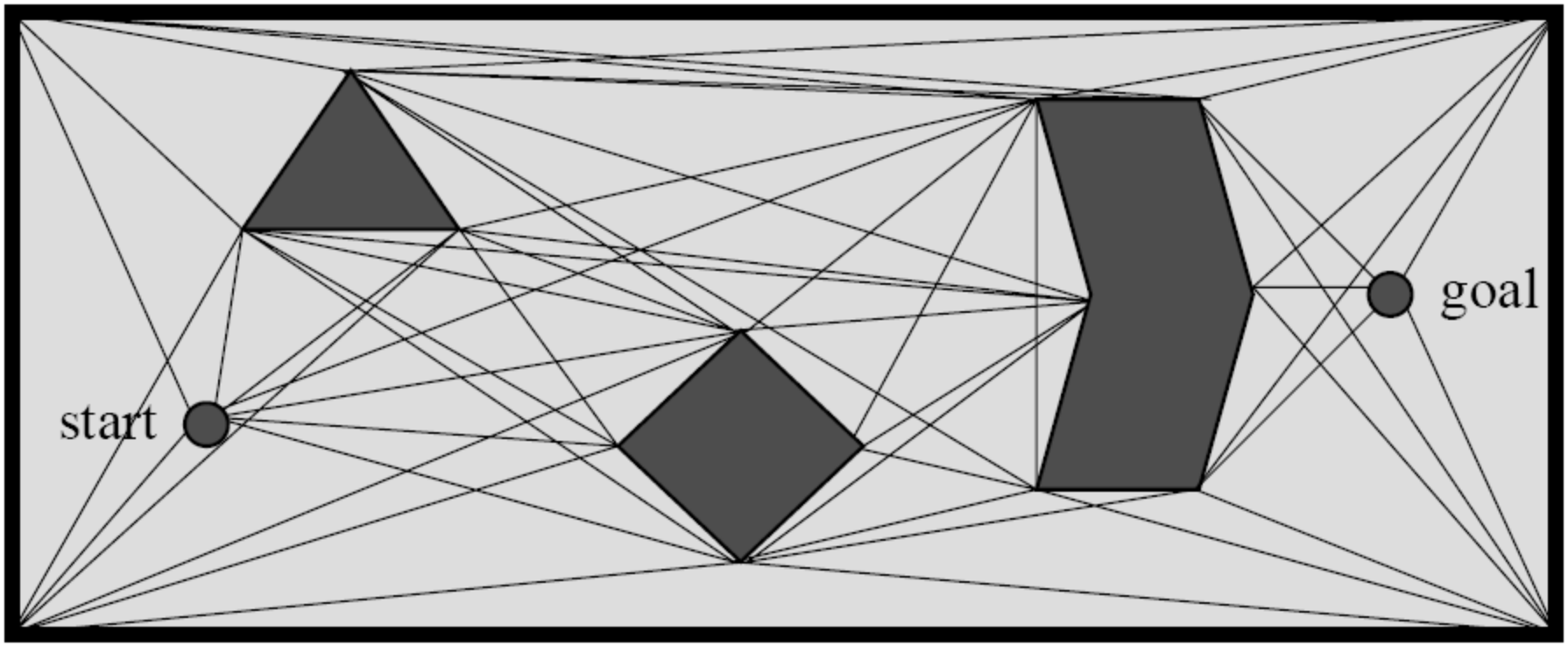
The Visibility Graph in Action (Part 4)

- Second, draw lines of sight from every vertex of every obstacle like before. Remember, lines along edges are also lines of sight.

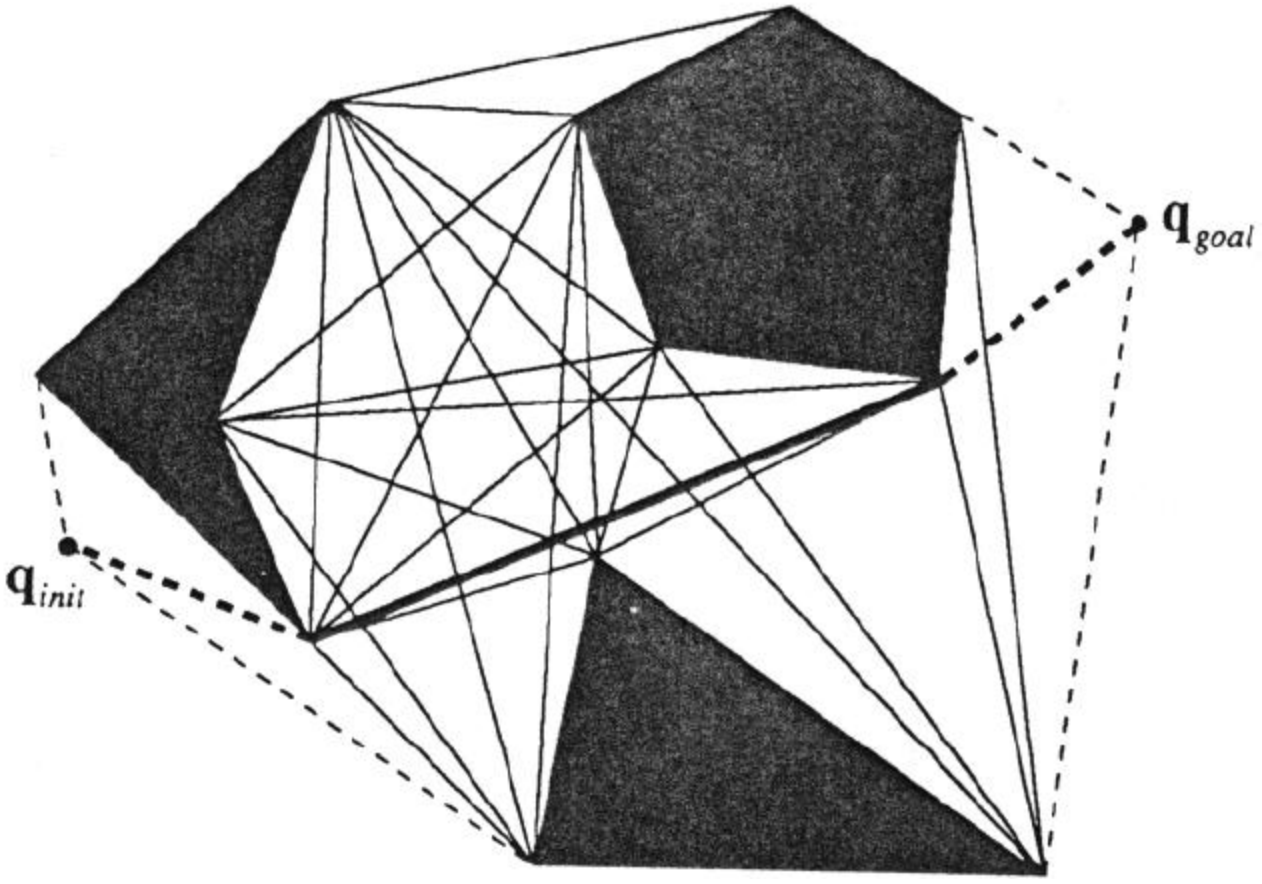


The Visibility Graph (Done)

- Repeat until you're done.



Visibility Graphs



The Sweepline Algorithm

- **Goal:** calculate the set of vertices v_i that are visible from v
- visibility: a segment v-v_i is visible if
  - it is not within the object
  - the closest line intersecting v-v_i is beyond v_i

Algorithm:

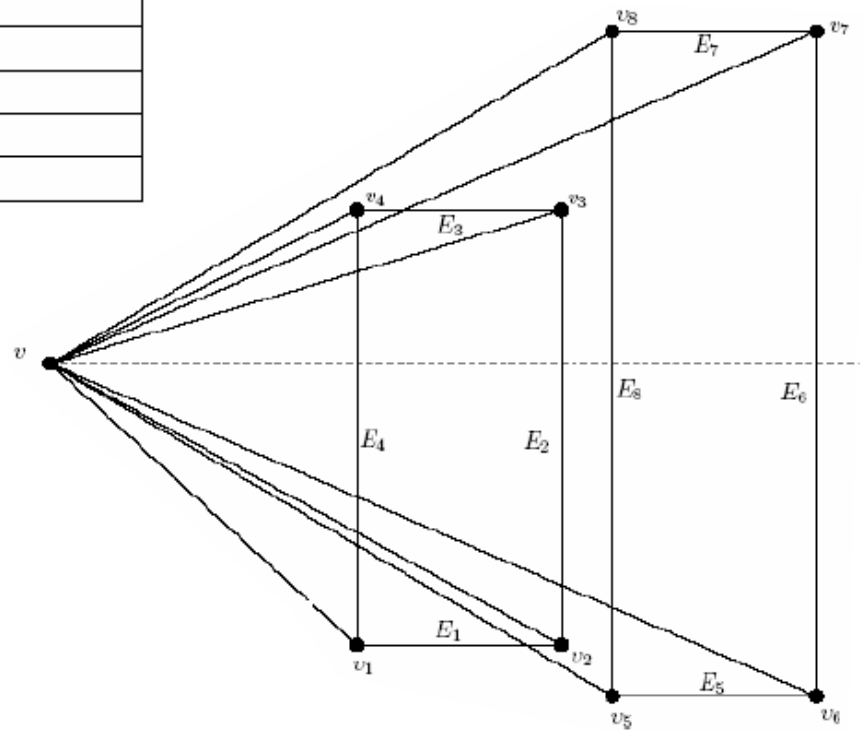
- Initially:
  - calculate the angle α_i of segment v-v_i and sort vertices by this creating list E
  - create a list of edges that intersect the horizontal from v sorted by intersection distance
- For each α_i
  - if v_i is visible to v then add v-v_i to graph
  - if v_i is the "beginning" of an edge E, insert E in S
  - if v_i is the "end" of and edge E, remove E from S

Analysis: For a vertex, n log n to create initial list, log n for each α_i

Overall: n log (n) (or n^2 log (n) for all n vertices)

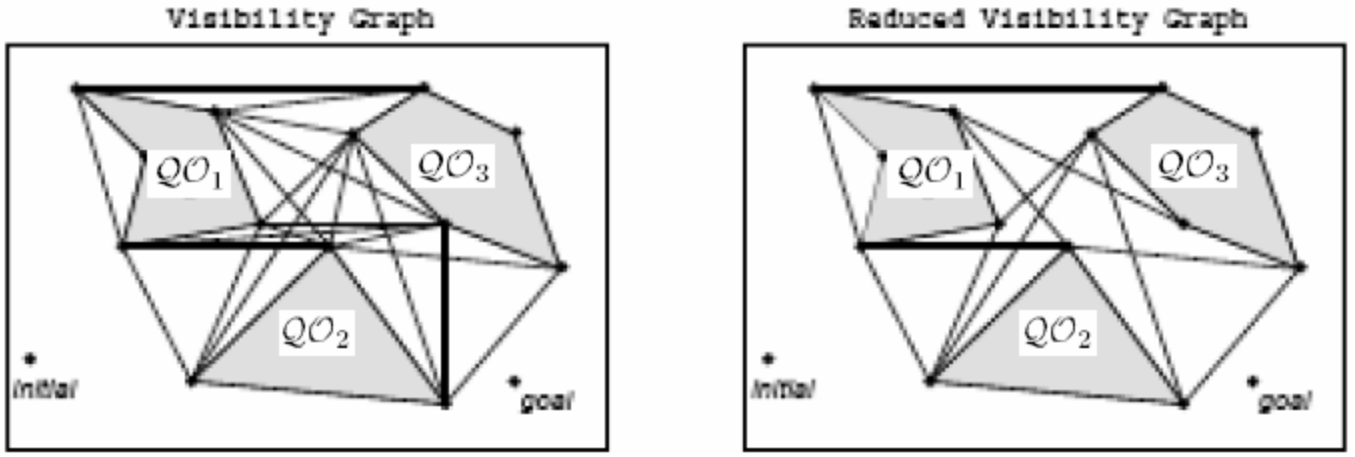
Example

Vertex	New $\mathcal{S}$	Actions
Initialization	$\{E_4, E_2, E_8, E_6\}$	Sort edges intersecting horizontal half-line
α_3	$\{E_4, E_3, E_8, E_6\}$	Delete E_2 from $\mathcal{S}$. Add E_3 to $\mathcal{S}$.
α_7	$\{E_4, E_3, E_8, E_7\}$	Delete E_6 from $\mathcal{S}$. Add E_7 to $\mathcal{S}$.
α_4	$\{E_8, E_7\}$	Delete E_3 from $\mathcal{S}$. Delete E_4 from $\mathcal{S}$. ADD (v, v_4) to visibility graph
α_8	$\{\}$	Delete E_7 from $\mathcal{S}$. Delete E_8 from $\mathcal{S}$. ADD (v, v_8) to visibility graph
α_1	$\{E_1, E_4\}$	Add E_4 to $\mathcal{S}$. Add E_1 to $\mathcal{S}$. ADD (v, v_1) to visibility graph
α_5	$\{E_4, E_1, E_8, E_5\}$	Add E_8 to $\mathcal{S}$. Add E_5 to $\mathcal{S}$.
α_2	$\{E_4, E_2, E_8, E_5\}$	Delete E_1 from $\mathcal{S}$. Add E_2 to $\mathcal{S}$.
α_6	$\{E_4, E_2, E_8, E_6\}$	Delete E_5 from $\mathcal{S}$. Add E_6 to $\mathcal{S}$.
Termination		



Reduced Visibility Graphs

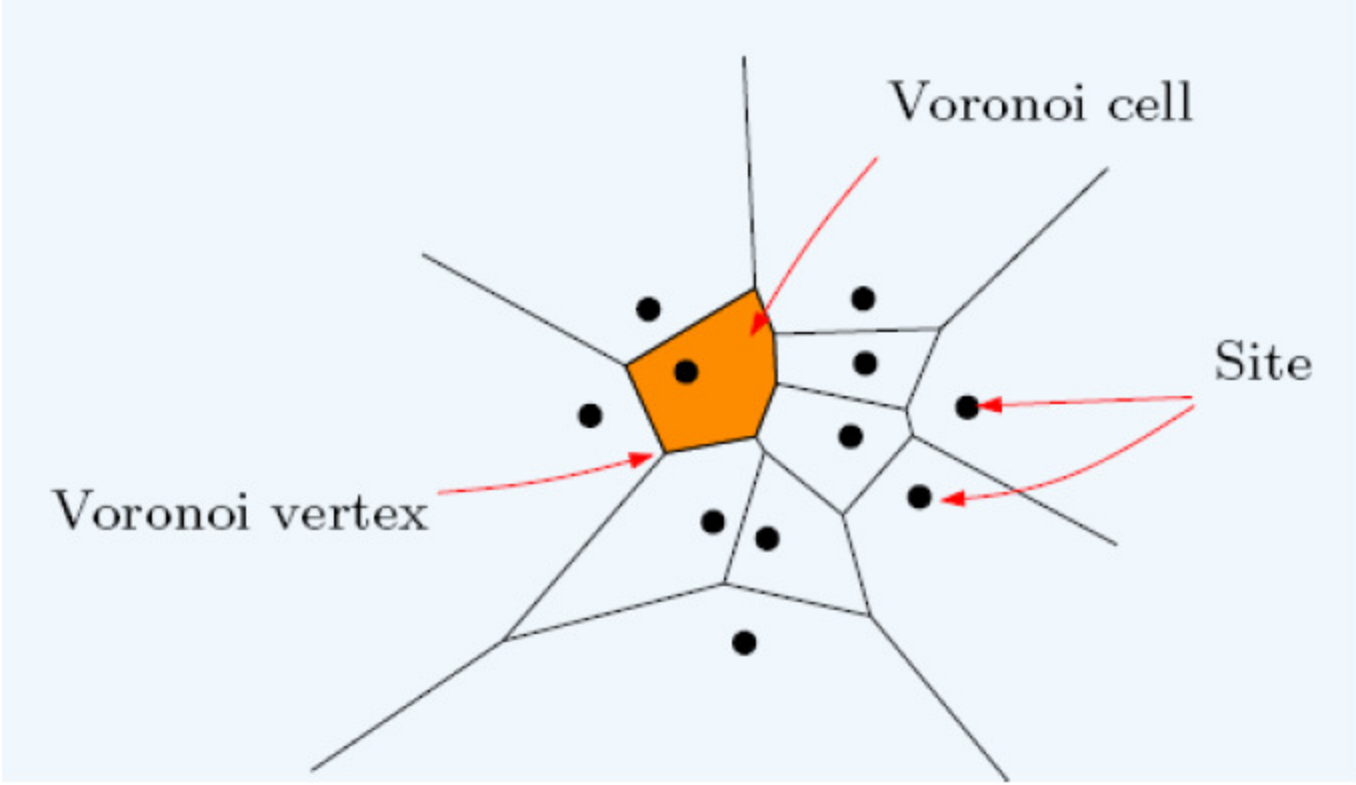
- The current graph as too many lines
  - lines to concave vertices
  - lines that “head into” the object
- A reduced visibility graph consists of
  - nodes that are convex
  - edges that are “tangent” (i.e. do not head into the object at either endpoint)



interestingly, this all only works in $\Re^2$

Deformation Retracts: Voronoi Diagrams

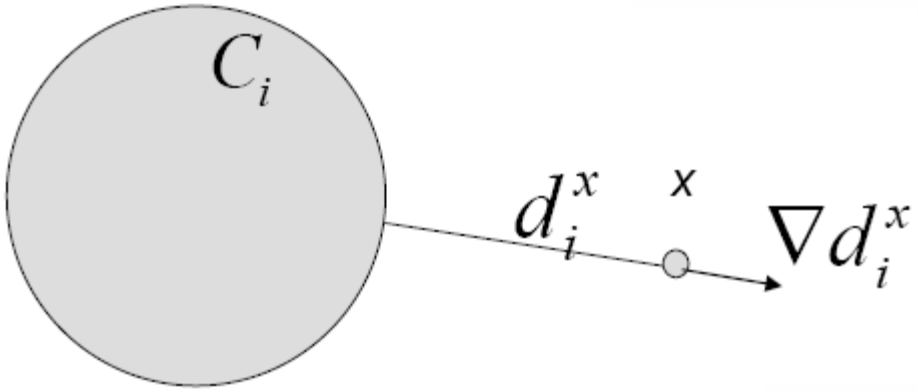
GVD: Set of points where the distance to two closest obstacles is the same



Beyond Points: Basic Definitions

Single-object distance function (Warning: convex obstacle)

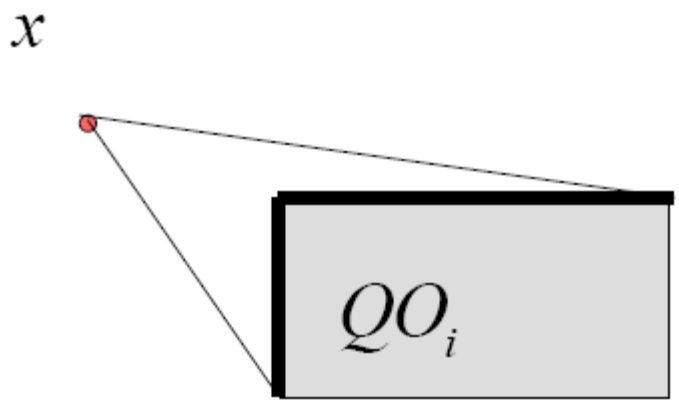
$$d_i^x(q) = \min_{c \in \mathcal{QO}_i} d(q, c)$$



$$\nabla d_i^x(q) = \frac{q - c}{d(q, c)}$$

Visible Distance Functions

Points within line of sight



$$\tilde{C}_i(x) = \{c \in QO_i : \forall t \in [0,1], x(1-t) + ct \in Q_{\text{free}}\}$$

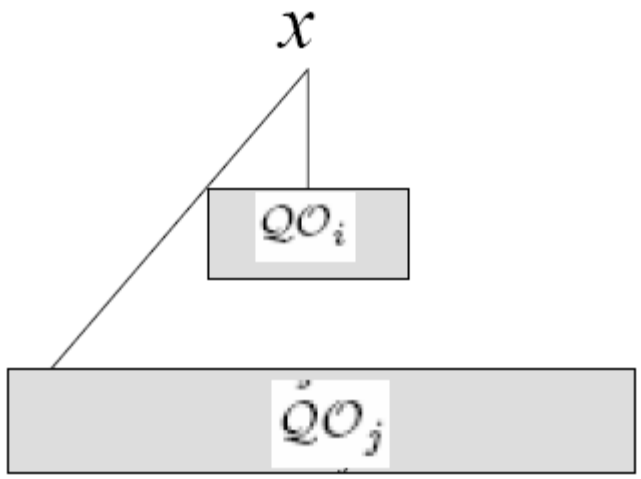
Visible Distance Functions

- *Single-object (Warning: convex obstacle)*

$$d_i(x) = \begin{cases} \text{distance} \quad \text{to } \mathcal{QO}_i & \text{if } c_i \in \tilde{C}_i(x) \\ \infty & \text{otherwise} \end{cases}$$

- *Multi-object*

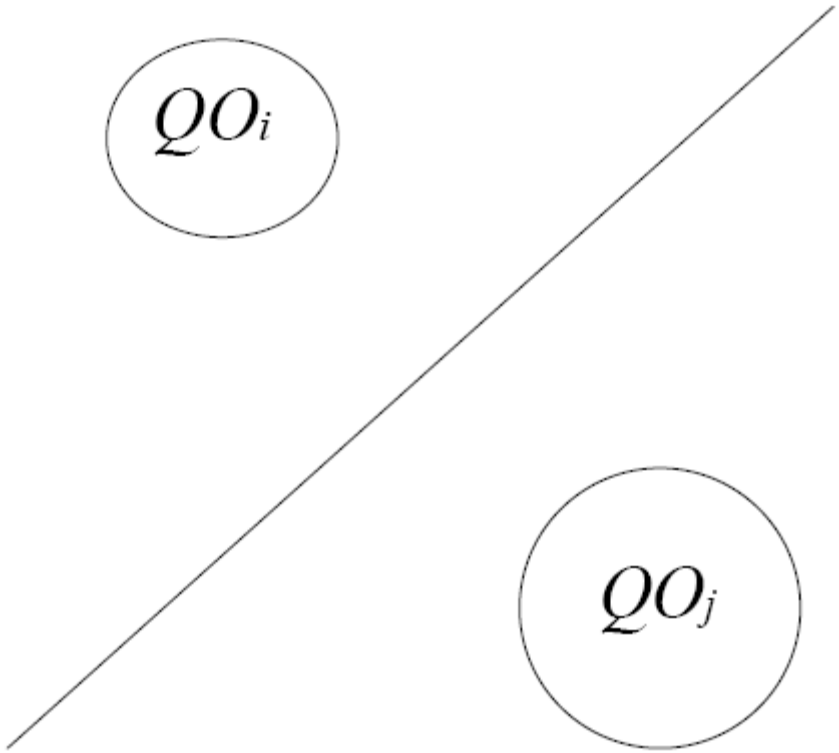
$$D(x) = \min_i d_i(x)$$



Two-Equidistant

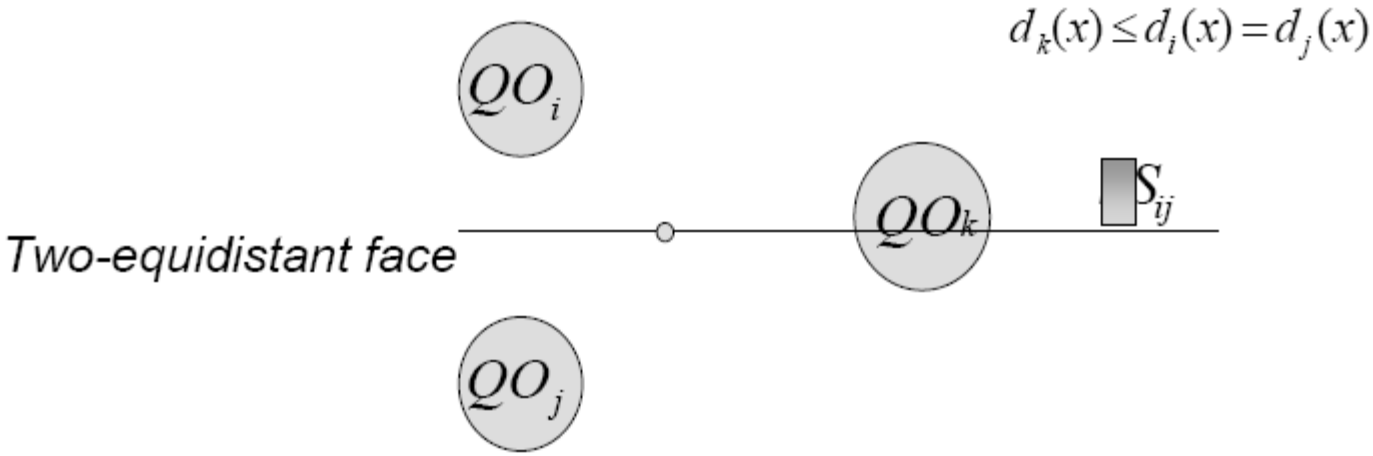
- *Two-equidistant surface*

$$S_{ij} = \{x \in Q_{\text{free}} : d_i(x) - d_j(x) = 0\}$$



More Rigorous Definition

- Going through obstacles

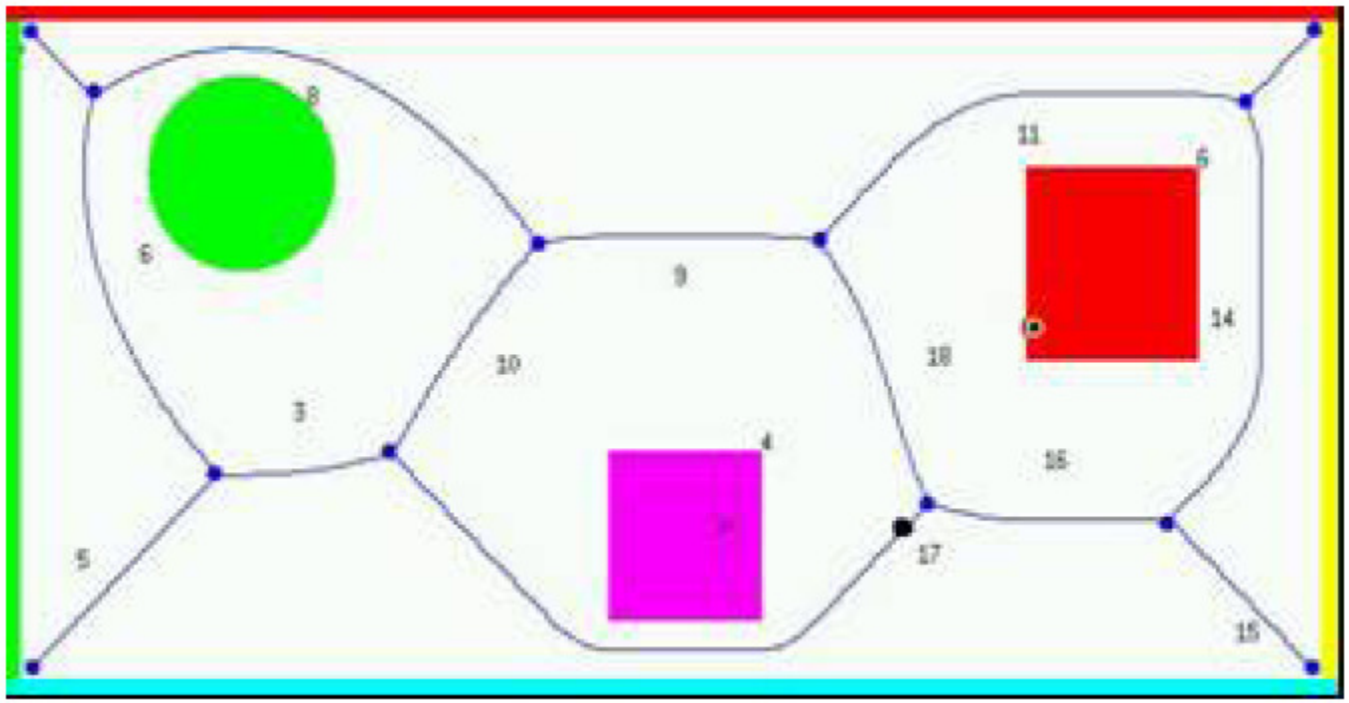


$$F_{ij} = \{x \in S_{ij} : d_i(x) = d_j(x) \le d_h(x), \forall h \neq i, j\}$$

Generalized Voronoi Diagram

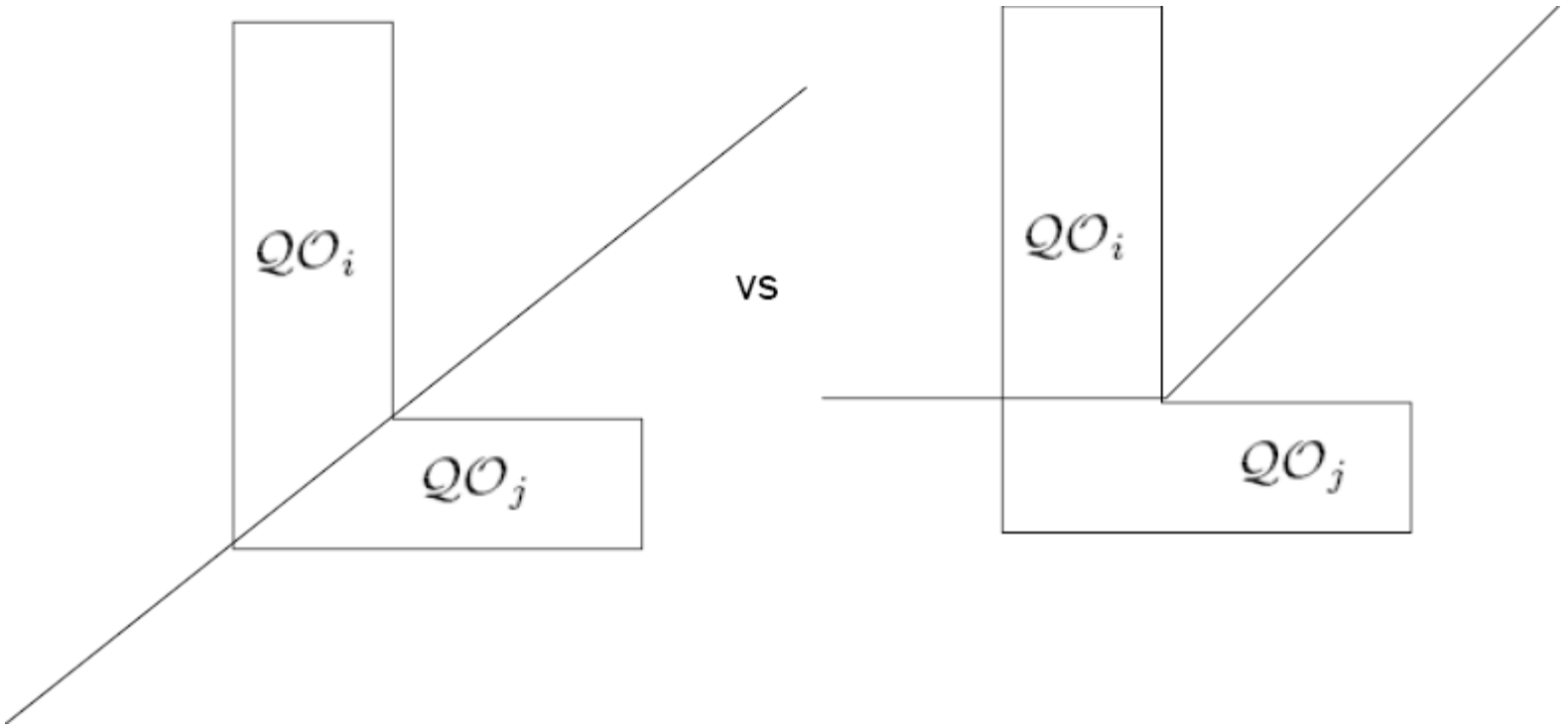
$$\mathrm{GVD} = \bigcup_{i=1}^{n-1} \bigcup_{j=i+1}^{n} F_{ij}$$

- In 2D, the GVD is one-dimensional
- It can also be shown to be a roadmap



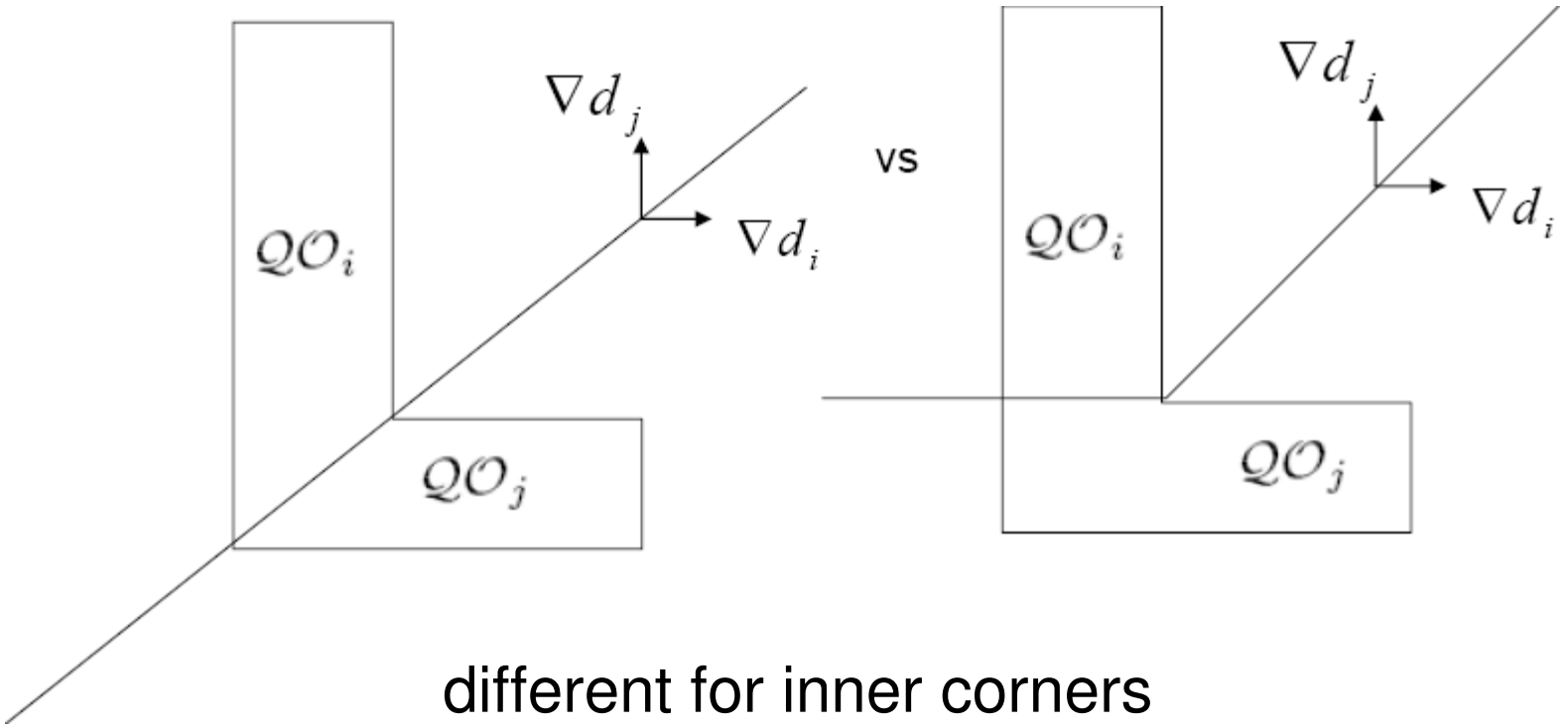
What about concave obstacles?

Suppose we represent concave obstacles by multiple convex obstacles:



What about concave obstacles?

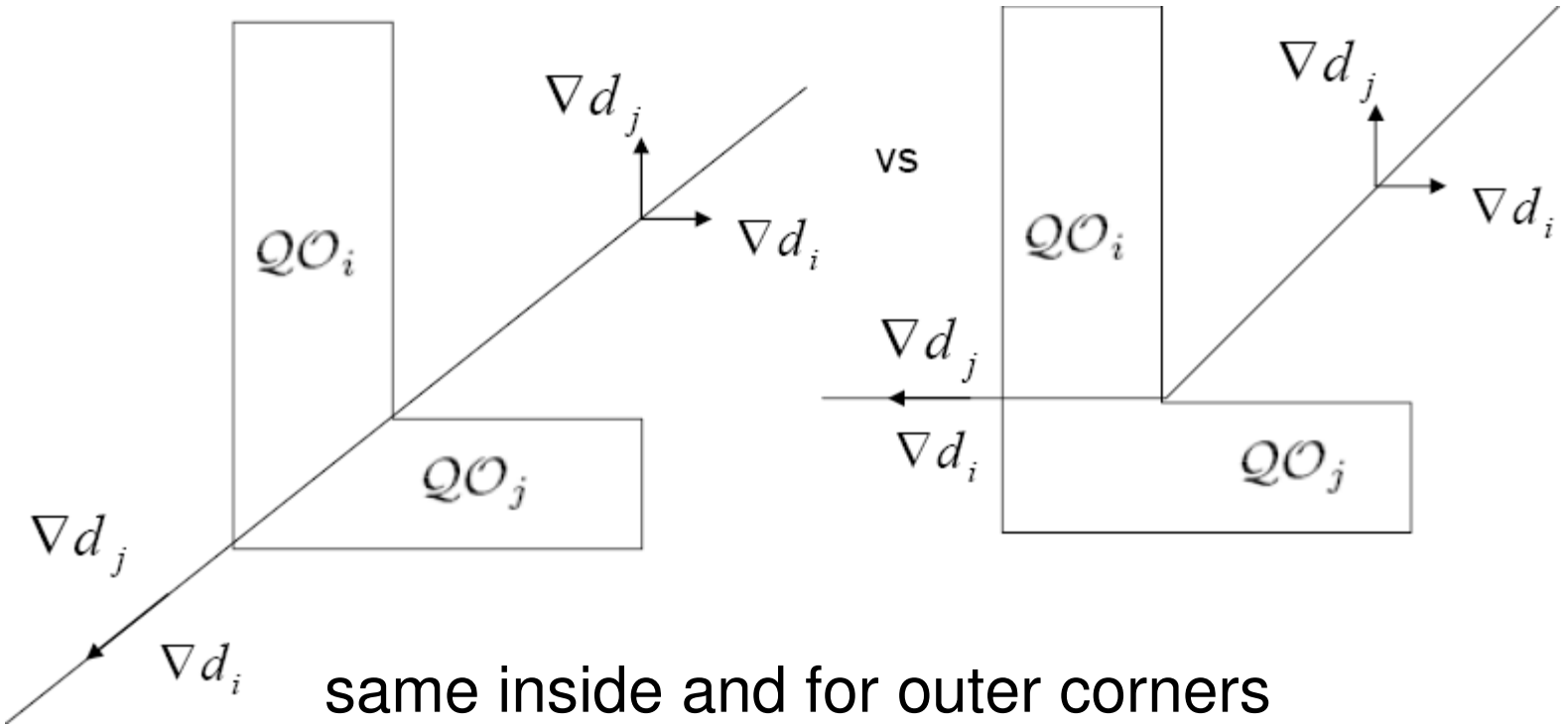
Consider individual distance gradients for convex regions...



different for inner corners

What about concave obstacles?

Consider individual distance gradients for convex regions...



same inside and for outer corners

Two-Equidistant: Final Definition

- *Two-equidistant surface*

$$S_{ij} = \{x \in Q_{\text{free}} : d_i(x) - d_j(x) = 0\}$$

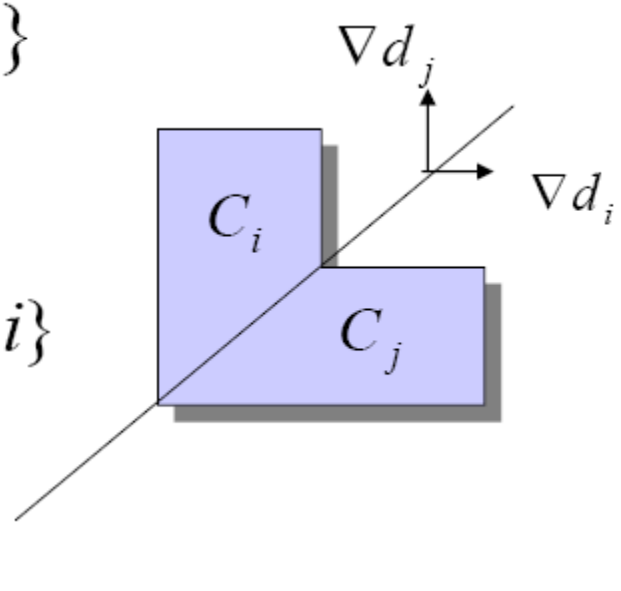
- Two-equidistant surjective surface

$$SS_{ij} = \{x \in S_{ij} : \nabla d_i(x) \neq \nabla d_j(x)\}$$

- Two-equidistant Face

$$F_{ij} = \{x \in SS_{ij} : d_i(x) \leq d_h(x), \forall h \neq i\}$$

$$\text{GVD} = \bigcup_{i=1}^{n-1} \bigcup_{j=i+1}^{n} F_{ij}$$

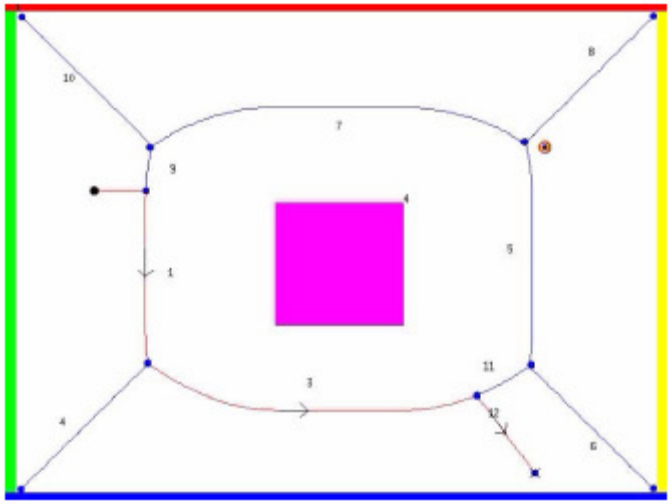


Curve Optimization Approach

- Homotopy Classes

$$f : U \to V \text{ and } g : U \to V$$
$$H : U \times [0,1] \to V$$
$$H(x,0) = f \text{ and } H(x,1) = g$$

$$[c] = \{\bar{c} \in C^0 \mid \bar{c} \sim c\}$$



- Deformation Retracts

$$A \subset X \text{ and } f : X \to A$$
$$H : X \times [0,1] \to X$$
$$H(x,0) = x \text{ and } H(x,1) \in A$$
$$\text{and } H(a,t) = a \text{ for } a \in A, t \in [0.1]$$

Pre-Image Theorem

$$f : R^m \rightarrow R^n$$

$$f^{-1}(c) = \{x \in R^m : f(x) = c\}$$

e.g. $f(x, y) = x^2 + y^2 \qquad f : R^2 \rightarrow R$

$f^{-1}(9)$ is a circle with radius 3

if $\forall x \in f^{-1}(c), Df(x)$ is full rank,

then $f^{-1}(c)$ is a manifold of dimension $m - n$

Proof for GVD Dimension

$$f(x) = d_i(x) - d_j(x), \quad f : R^m \to R, \quad c = 0$$

$$(d_i - d_j)^{-1}(0) \text{ s.t. } D(d_i - d_j) \text{ is full rank}$$

$$\Rightarrow \dim((d_i - d_j)^{-1}(0)) = m - n = m - 1$$

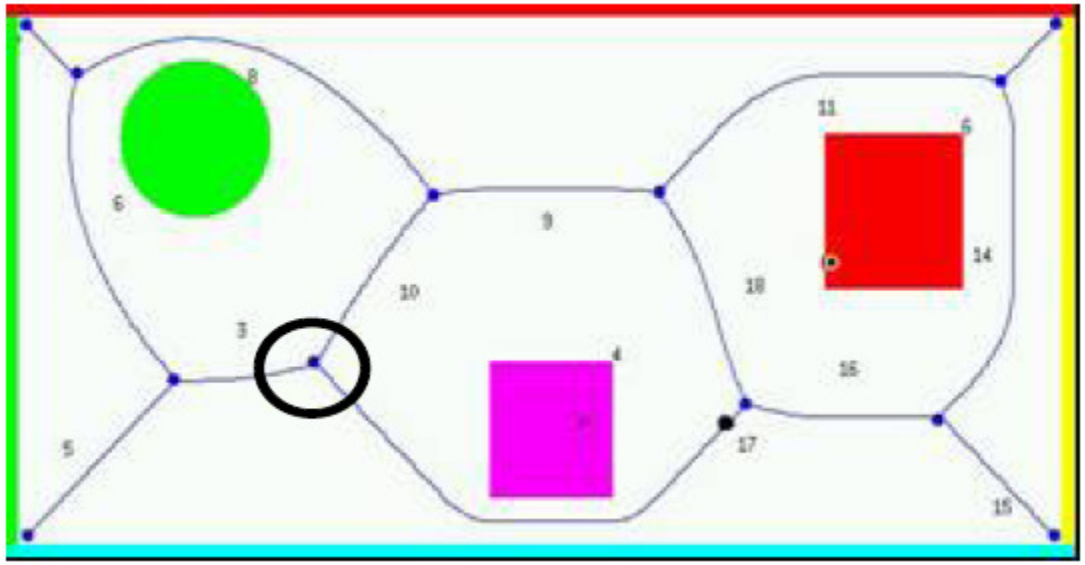
$$\text{Show } D(d_i - d_j) \text{ is full rank} \Leftrightarrow$$

$$D(d_i - d_j) \text{ is not a 0 row vector}$$

$$\nabla d_i \neq \nabla d_j \Leftrightarrow \nabla d_i - \nabla d_j \neq 0 \Leftrightarrow \nabla(d_i - d_j) \neq 0 \Leftrightarrow D(d_i - d_j)$$

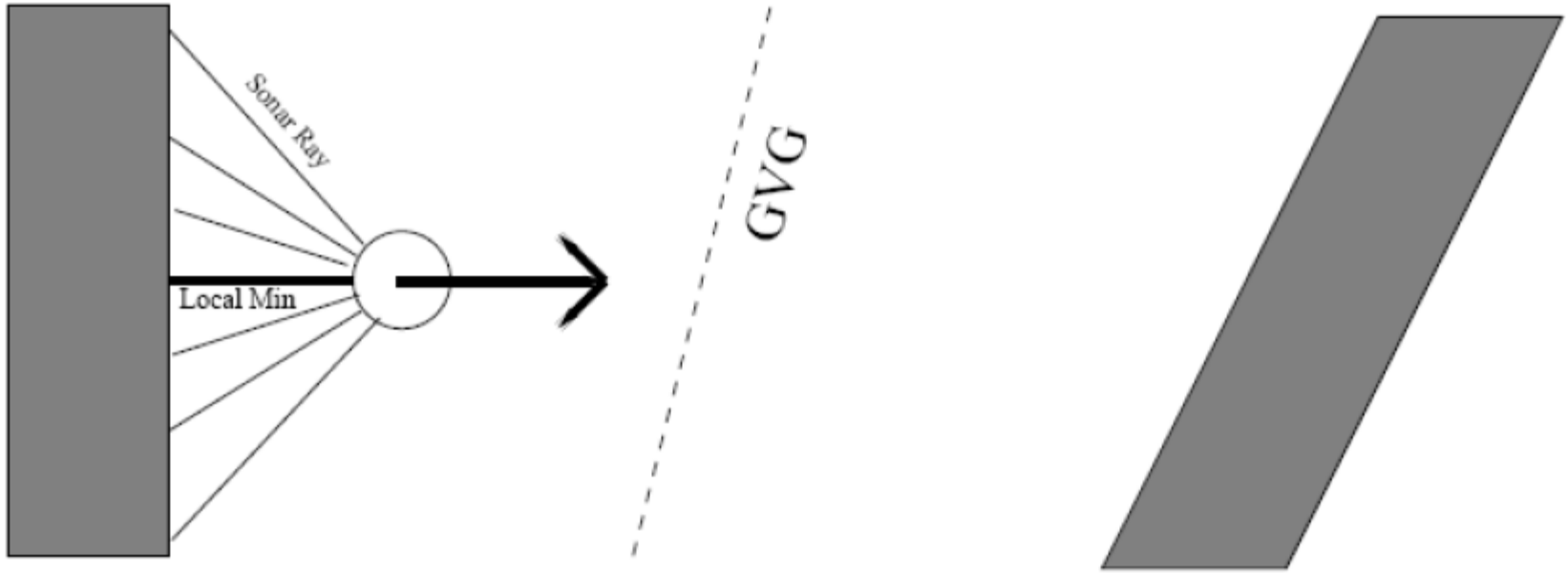
More on GVD...

- Is the GVD a 1-Dimension manifold?



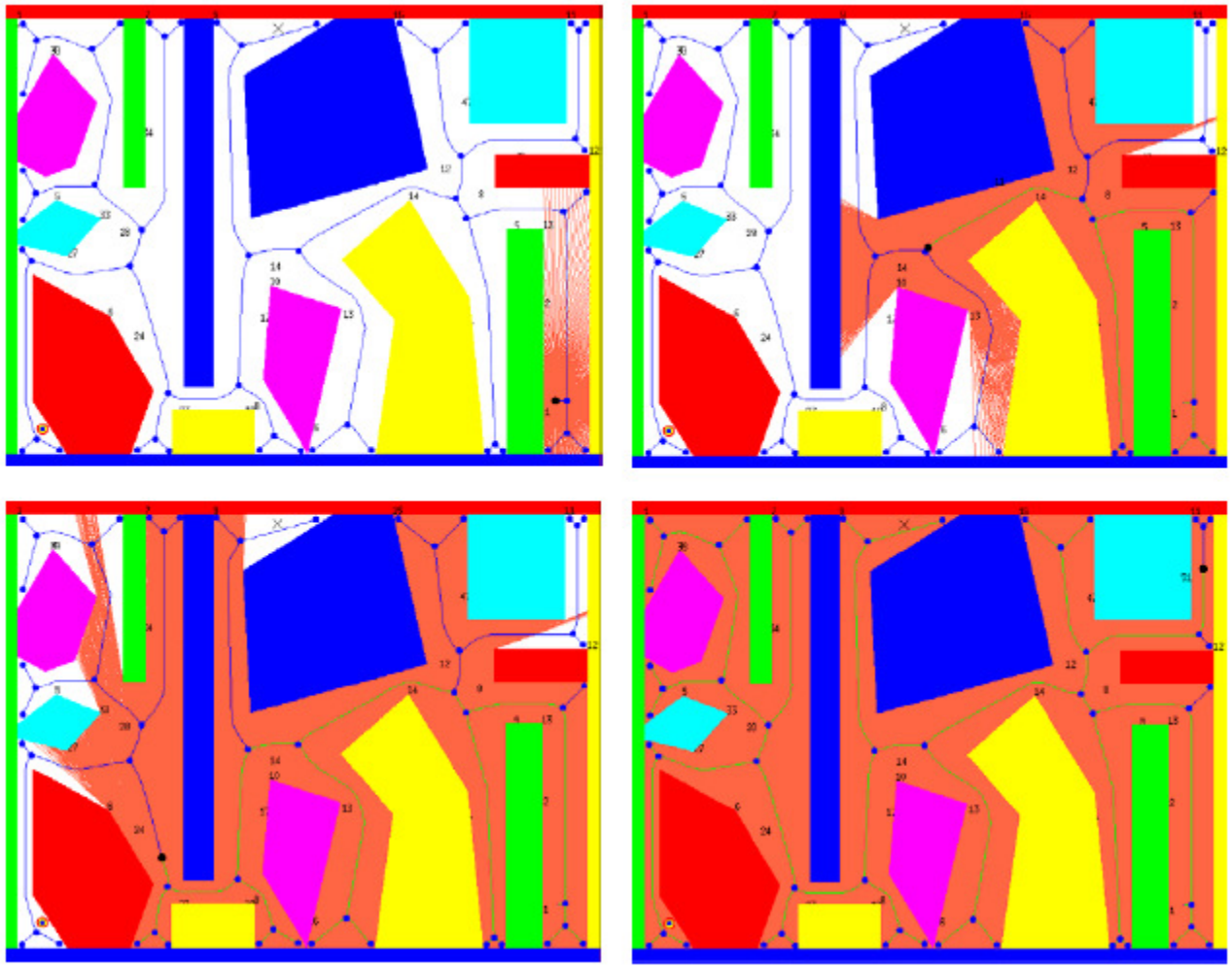
No, but it's the union of 1D manifolds

Accessibility (in the Plane)



Departability

$$\forall \; q \in \mathcal{Q}_{\text{free}}, \quad \exists q' \in \text{ GVD such that } \; sq + (1-s)q' \in \mathcal{Q}_{\text{free}} \quad \forall s \in [0,1]$$



GVD Connected?

- Proof:

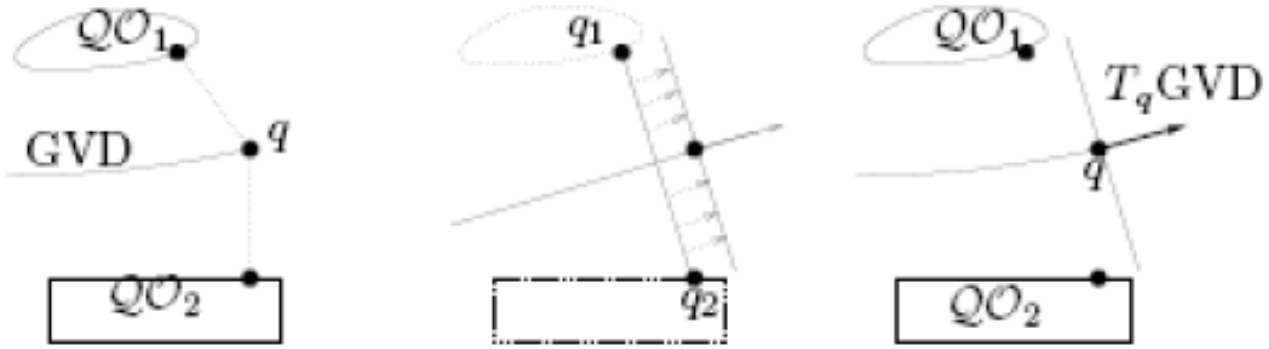
$$\mathrm{Im} : Q_{\mathrm{free}} \rightarrow GVD$$

- Im is continuous (Prof. Yap, NYU)
- Im of a connected set, under a continuous map, is a connected set

$$\because \text{for each connected component of } Q_{\mathrm{free}},$$
$$GVD \text{ is connected.}$$

Traceability in the Plane

Tangent



Pass a line through two closest points on two closest obstacles

Orthogonal is tangent

Correction

$$y^{k+1} = y^k - (\nabla_y G)^{-1} G(y^k, \lambda^k)$$

Continuous Control Laws

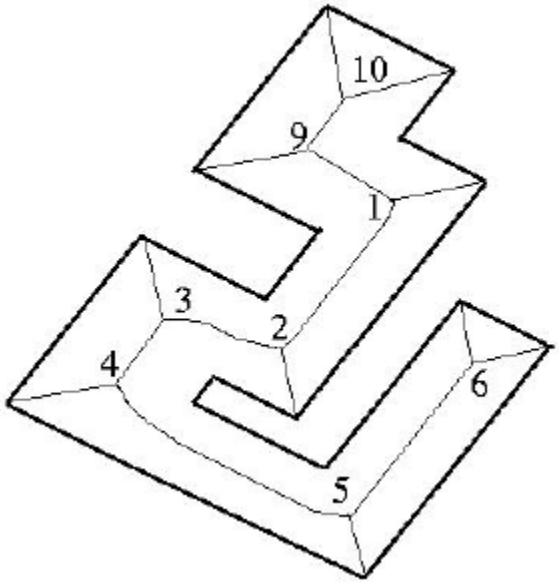
Edge : $G(x) = 0 = d_i(x) - d_j(x)$

$$\dot{x} = \alpha \, \text{Null}(\nabla G(x)) + \beta (\nabla G(x))^{\dagger} G(x)$$

where

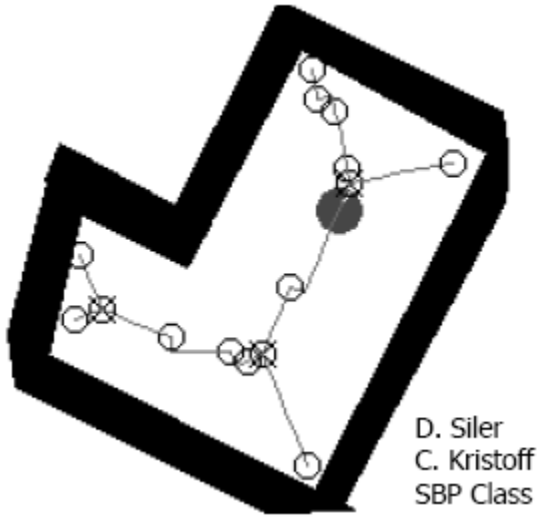
- α and β are scalar gains
- Null($\nabla G(x)$) is the null space of $\nabla G(x)$
- $(\nabla G(x))^{\dagger}$ is the Penrose pseudo inverse of $\nabla G(x)$, i.e.,

$$(\nabla G(x))^{\dagger} = (\nabla G(x))^{T} (\nabla G(x)(\nabla G(x))^{T})^{-1}$$



Meet Point : $G(x) = 0 = \begin{Bmatrix} d_i(x) - d_j(x) \\ d_i(x) - d_k(x) \end{Bmatrix}$

$$\dot{x} = \alpha \, \text{Null}(\nabla G(x)) + \beta (\nabla G(x))^{\dagger} G(x)$$



D. Siler
C. Kristoff
SBP Class

Algorithm for exploration

- Trace an edge until reach a meet point or a boundary point
- If a boundary point, return to the previous meet point, otherwise pick a new edge to trace
- If all edges from meet point are already traced, search the graph for a meet point with untraced edges
- When all meet points have no untraced edges, complete.

Generalized Voronoi Graph

- In 3-Dimensions

$$F_{ijk} = F_{ij} \cap F_{ik} \cap F_{jk}$$

$$S_{ij} = \{x \in Q_{\text{free}} : d_i(x) - d_j(x) = 0\}$$

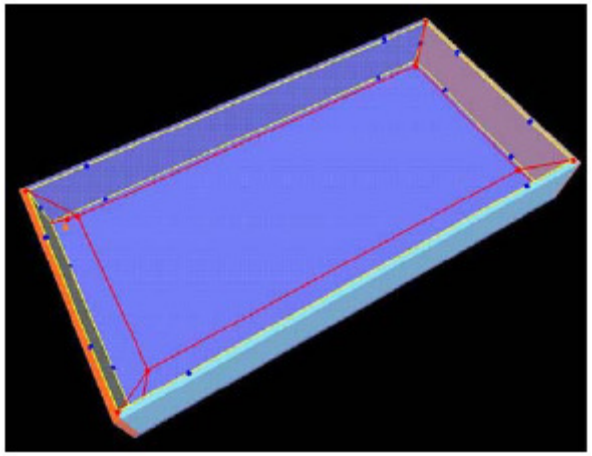
$$SS_{ij} = \{x \in S_{ij} : \nabla d_i(x) \neq \nabla d_j(x)\}$$

$$F_{ij} = \{x \in SS_{ij} : d_i(x) \leq d_h(x), \forall h \neq i\}$$

- In *m*-Dimensions

$$F_{ijk\ldots m} = F_{ij} \cap F_{ik} \ldots \cap F_{im}$$

$$= F_{ij\ldots m-1} \cap F_{im}$$



GVD vs. GVG

	Equidistant (#obs)	Dim	Codim
GVD	2	*m-1*	1
GVG	*m*	1	*m-1*

Proof for GVG Dimension

- For 3-Dimensions

$$f = \begin{pmatrix} d_i - d_j \\ d_i - d_k \end{pmatrix}, \quad f : R^3 \to R^2$$

$$f^{-1}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = f^{-1}(0)$$

$$D\begin{pmatrix} d_i - d_j \\ d_i - d_k \end{pmatrix} \neq 0, \ \text{since} \ \nabla d_i \neq \nabla d_j, \nabla d_i \neq \nabla d_k$$

Proof for GVG Dimension

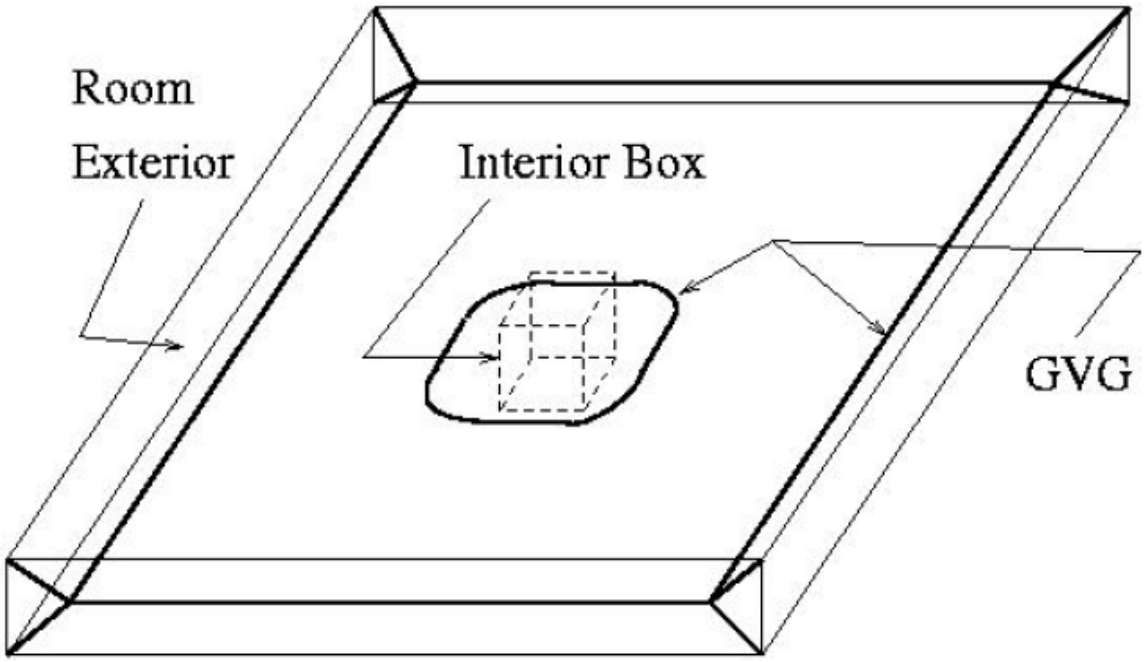
- For *m*-Dimensions

$$f : \begin{pmatrix} d_{i_1} - d_{i_2} \\ \cdot \\ \cdot \\ \cdot \\ d_{i_1} - d_{i_m} \end{pmatrix}, \text{where } f : R^m \rightarrow R^{m-1}$$

$$\text{By Pre - Image Theorem, } \dim(f^{-1}) = m - (m-1) = 1$$

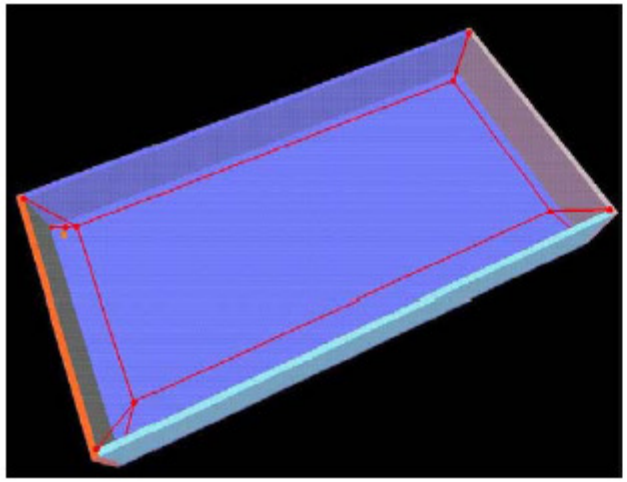
Remaining questions for GVGs

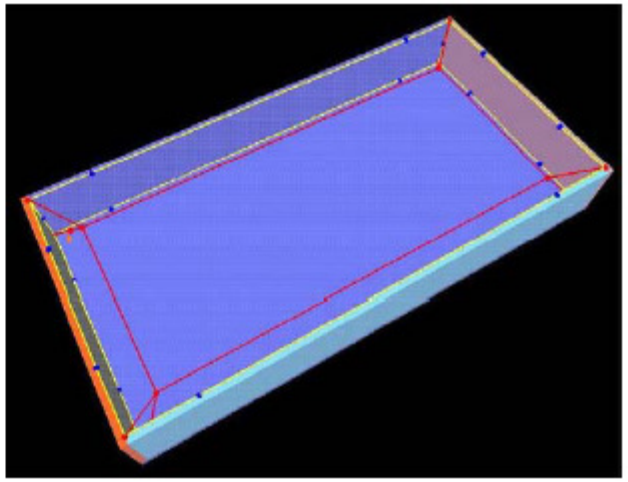
- Traceability, Accesibility, Departability?
- Connectivity?

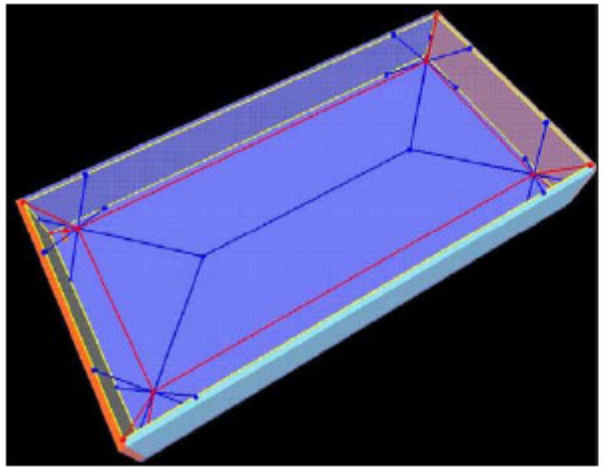


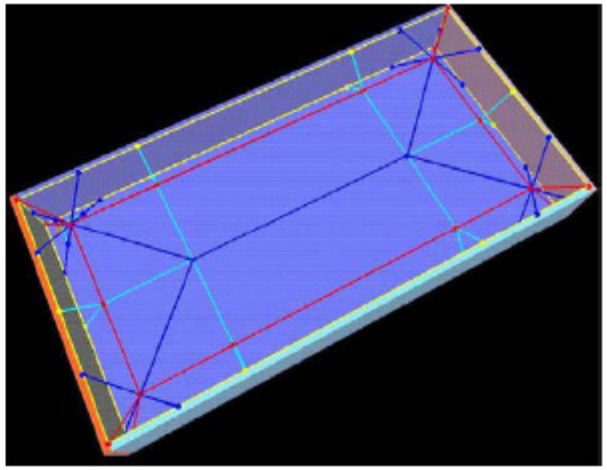
- More needs to be done...

Some examples

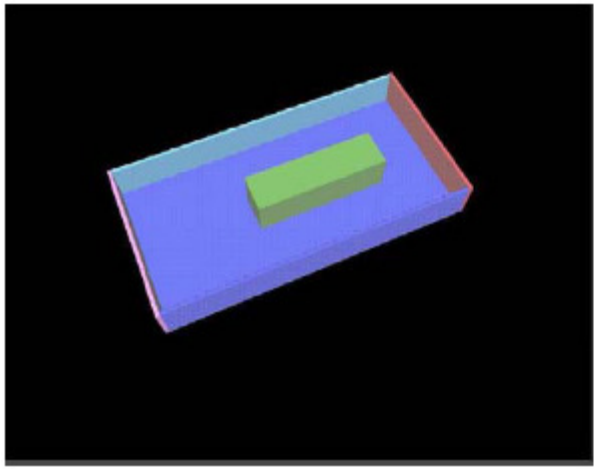


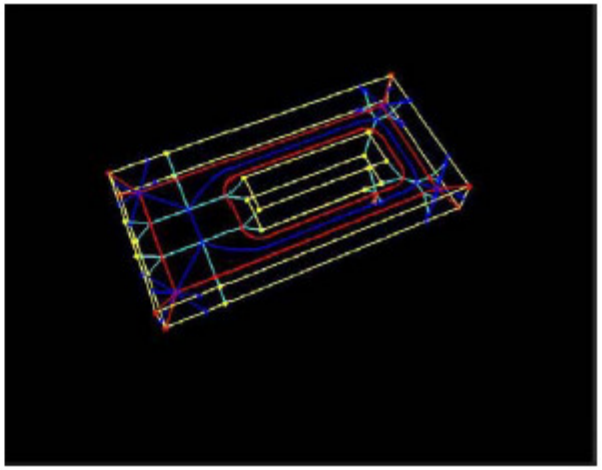


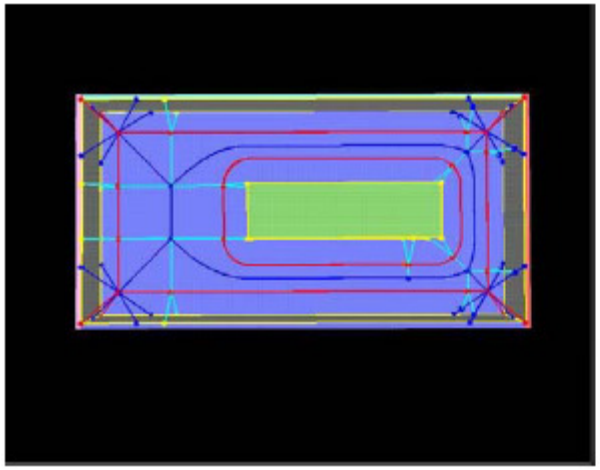


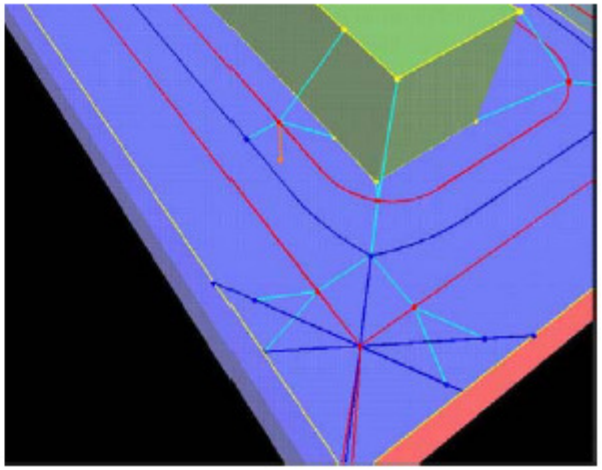


Room With Box

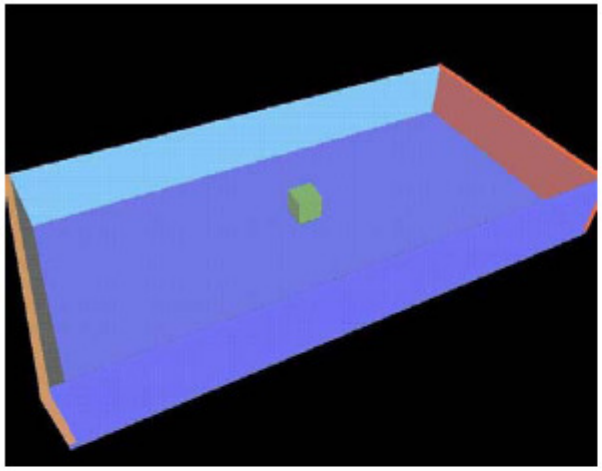


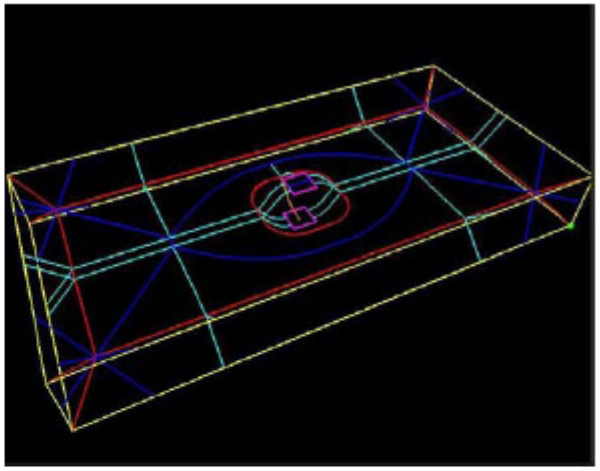


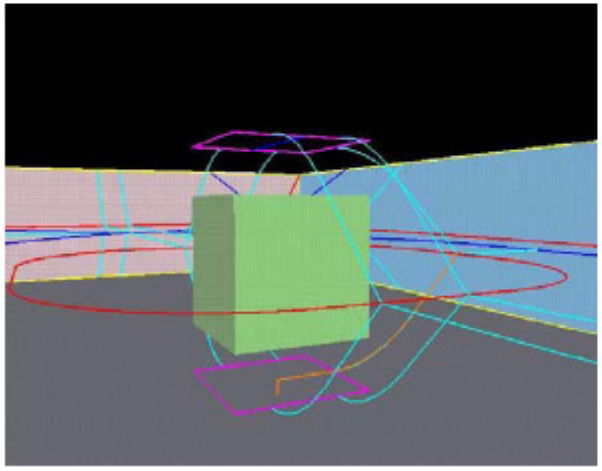


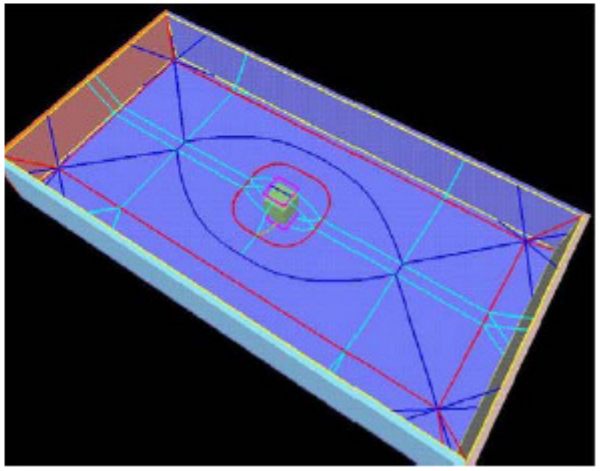


Floating Box

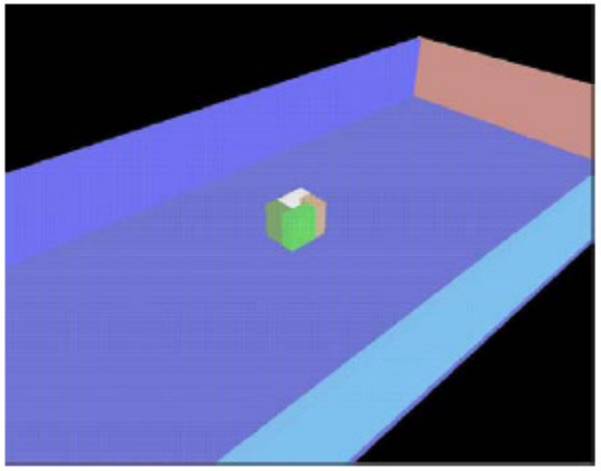


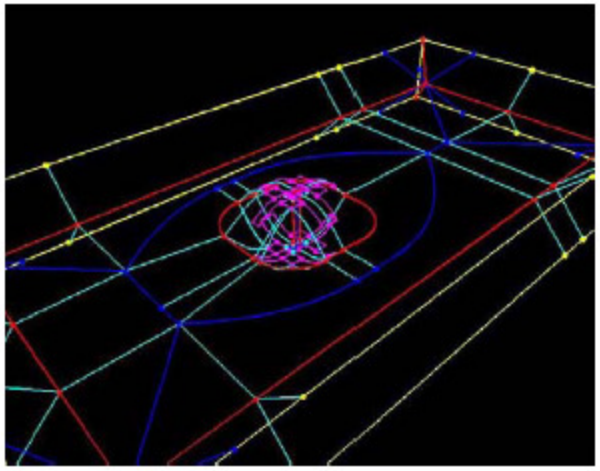


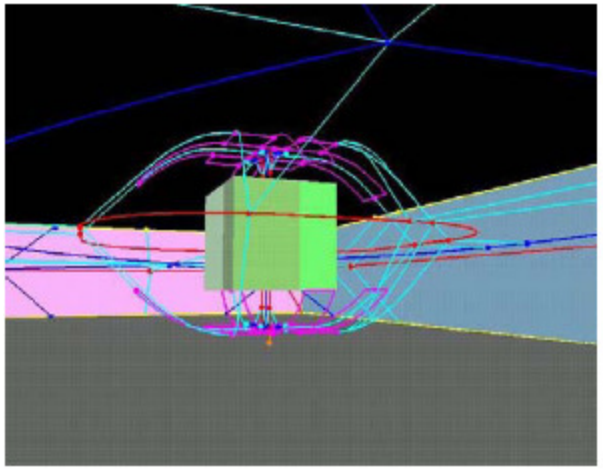


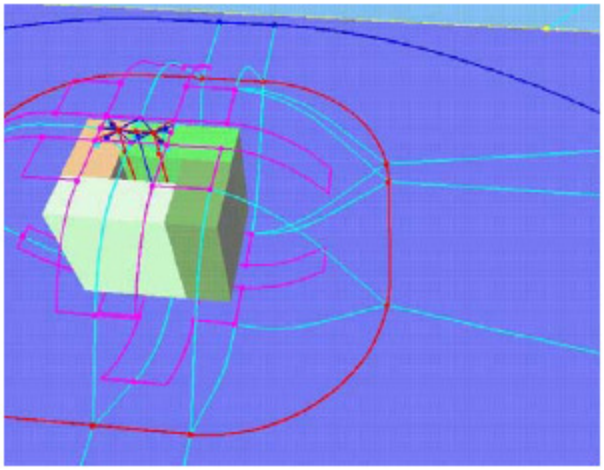


Box with Opening

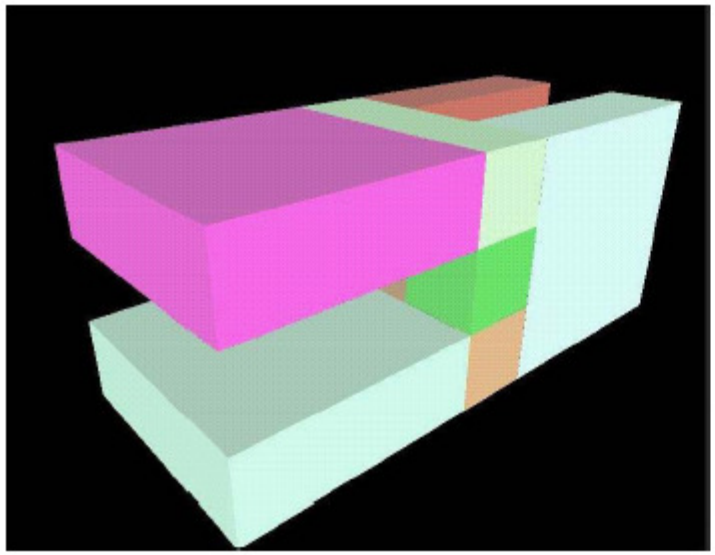


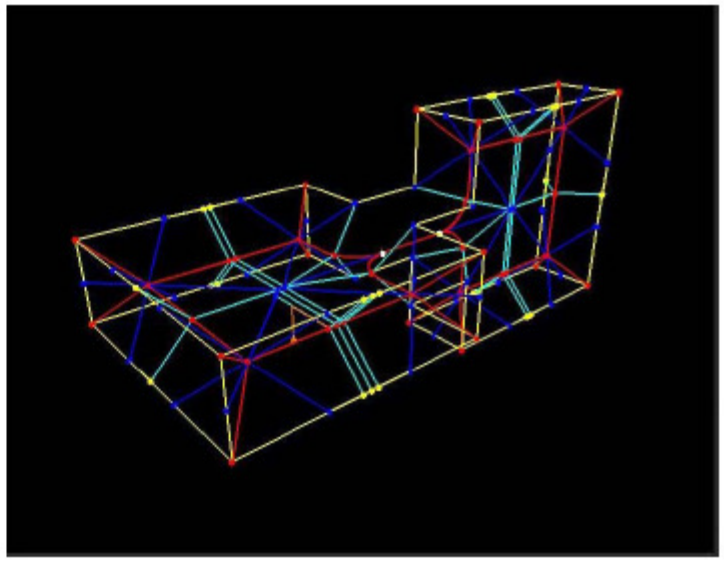






More Complicated Environments





Topological Maps (Kuipers)

- Topological map represents spatial properties of actions and of places and paths in the environment. Topological map is defined as the minimal models of an axiomatic theory describing the relationship between the different sources of information explained by map (Remolina and **Kuipers**, Artificial Intelligence, 2003)
- Topological maps represent the world as a graph of places with the arcs of the graph representing movements between places (Kortenkamp & Weymouth, AAAI-94)
- Topological maps represent the robot environment as graphs, where nodes corresponds to distinct places, and arcs represent adjacency. A key advantage of topological representations is their compactness (Thrun, et at. 1999)
- Topological localization uses a graph representation that captures the connectivity of a set of features in the environment (Radhakrishnan & Nourkbash, IROS 1999)

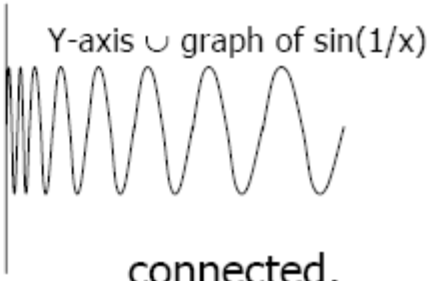
Topology (Really, Connectivity)

- A ***topology*** is a collection ***T*** of subsets of ***X***
  - $\varnothing, X \in T, \quad a_1 \cup a_2 \cup \ldots \in T, \quad a_1 \cap a_2 \cap \ldots \cap a_n \in T$
  - *What is the relevance?*

- ***path connected***

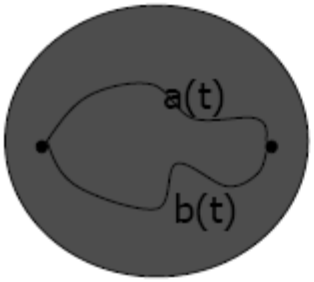
not connected

Connected,
path connected

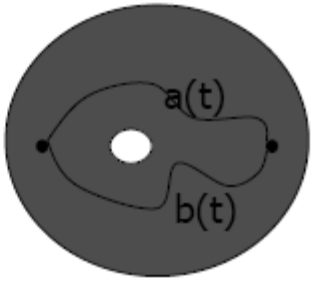


connected,
but not path connected

- ***simply connected (contractible)***



simply connected

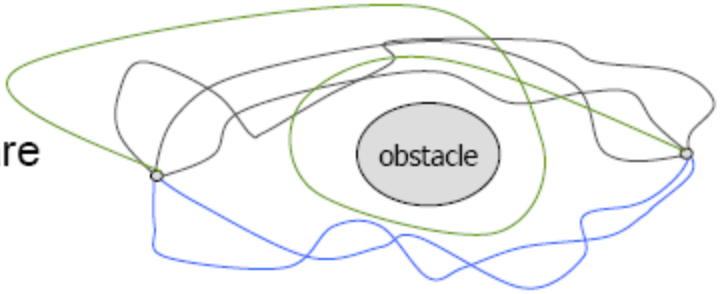


not simply
connected

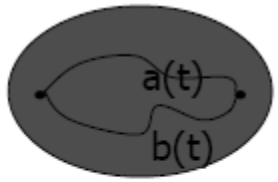
Homotopy

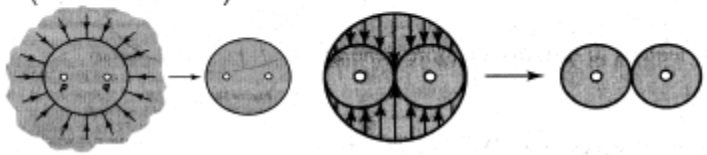
- Two paths ***f, f' : [0,1] → X***, are ***path-homotopic***
  F(s,0)=f(s) and ***F(s,1)=f'(s)***;
  F(0,t)=x_0 and ***F(1,t)=x_1*** $\forall \in$s [0,1], $\forall \in$t [0,1]

Path-Homotopy class [f] set of the mappings that are path-homotopic to f.

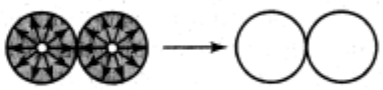


- ***Fundamental group π_1*** : set of path-homotopy classes
  - ***X*** is **simply connected** if π_1 is the trivial (one-element)

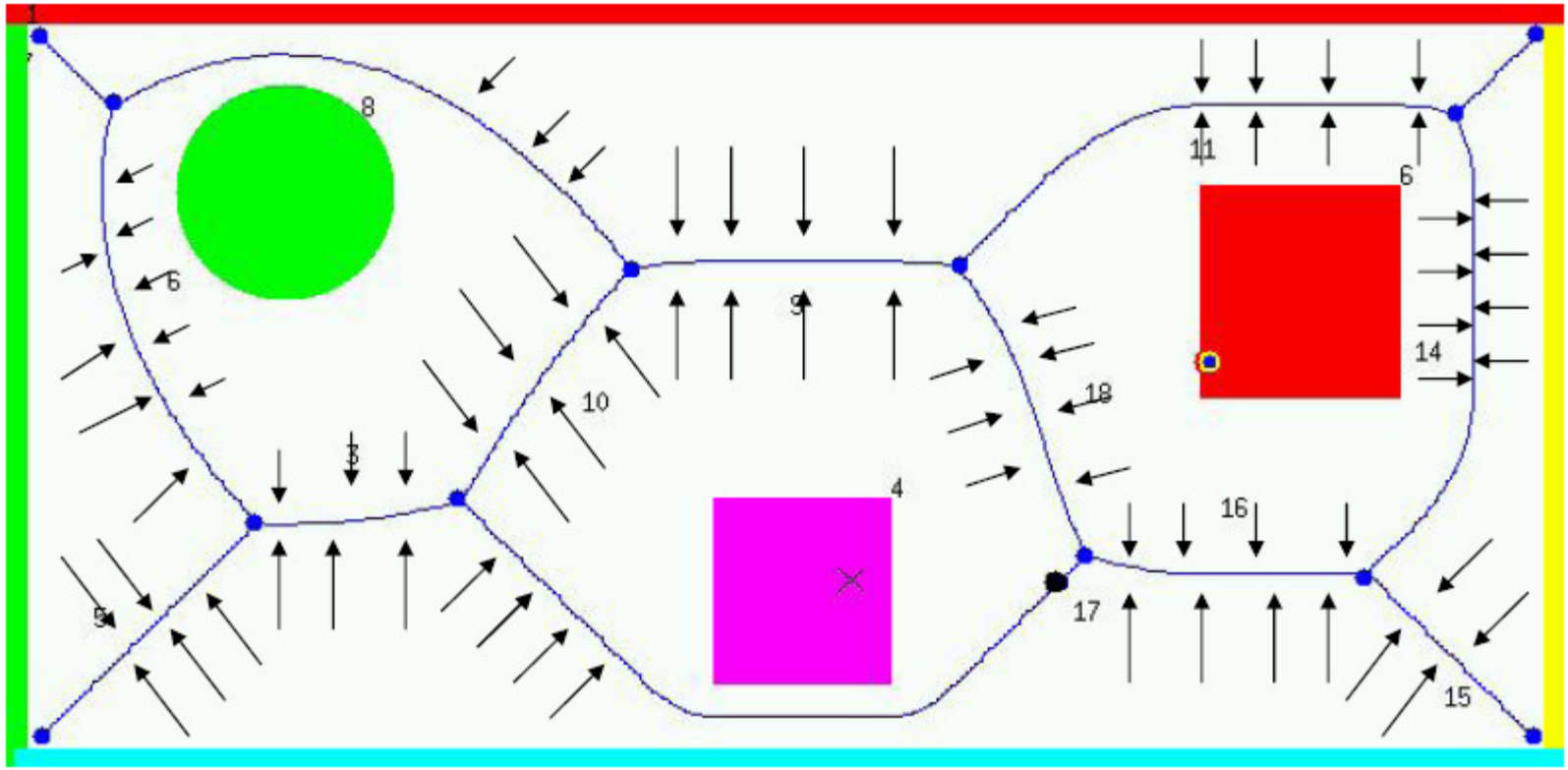




- ***A*** is a ***deformation retract*** iff ***H:X×[0,1]→X***,
  H(x,0)=x and ***H(x,1)*** $\in$***A*** $\forall$***x*** $\in$***X***, and ***H(a,t)=a*** $\forall$***a*** $\in$***A***, ***t*** $\in$***[0,1]***.
  - H is called a **deformation retraction**

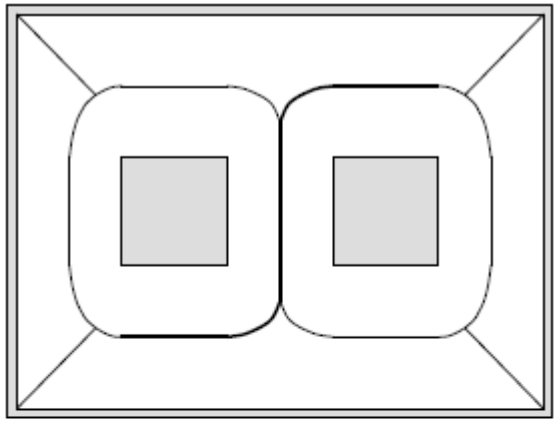


Deformation Retraction: GVG in Plane

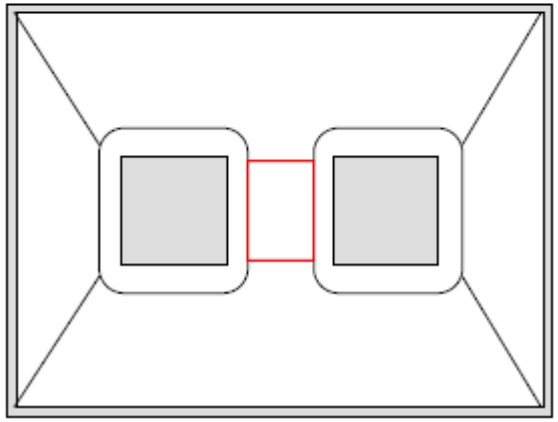


Topological Map: Good and Bad

- Topological Map: *For each homotopy class in free space, there is a corresponding homotopy class in the map.*



- Good Topological map : the first fundamental groups have the same cardinality

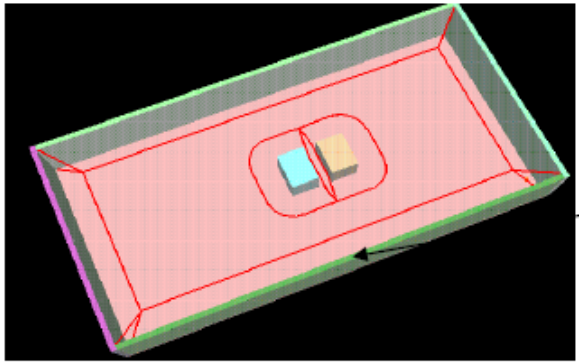


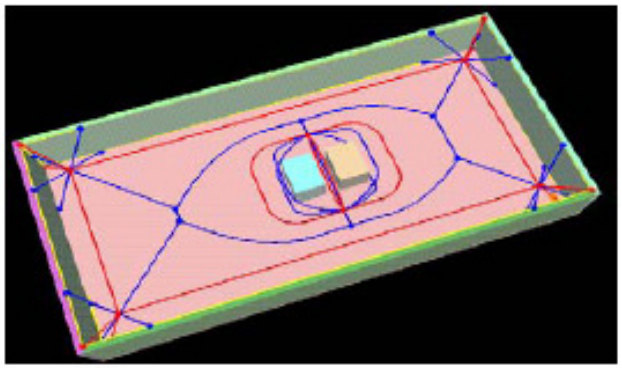
- Bad Topological map : redundant homotopy classes in the map

Bad Topological Maps in Higher Dims

- In general, there cannot be a one-dimensional deformation retract in a space with dimension greater than two

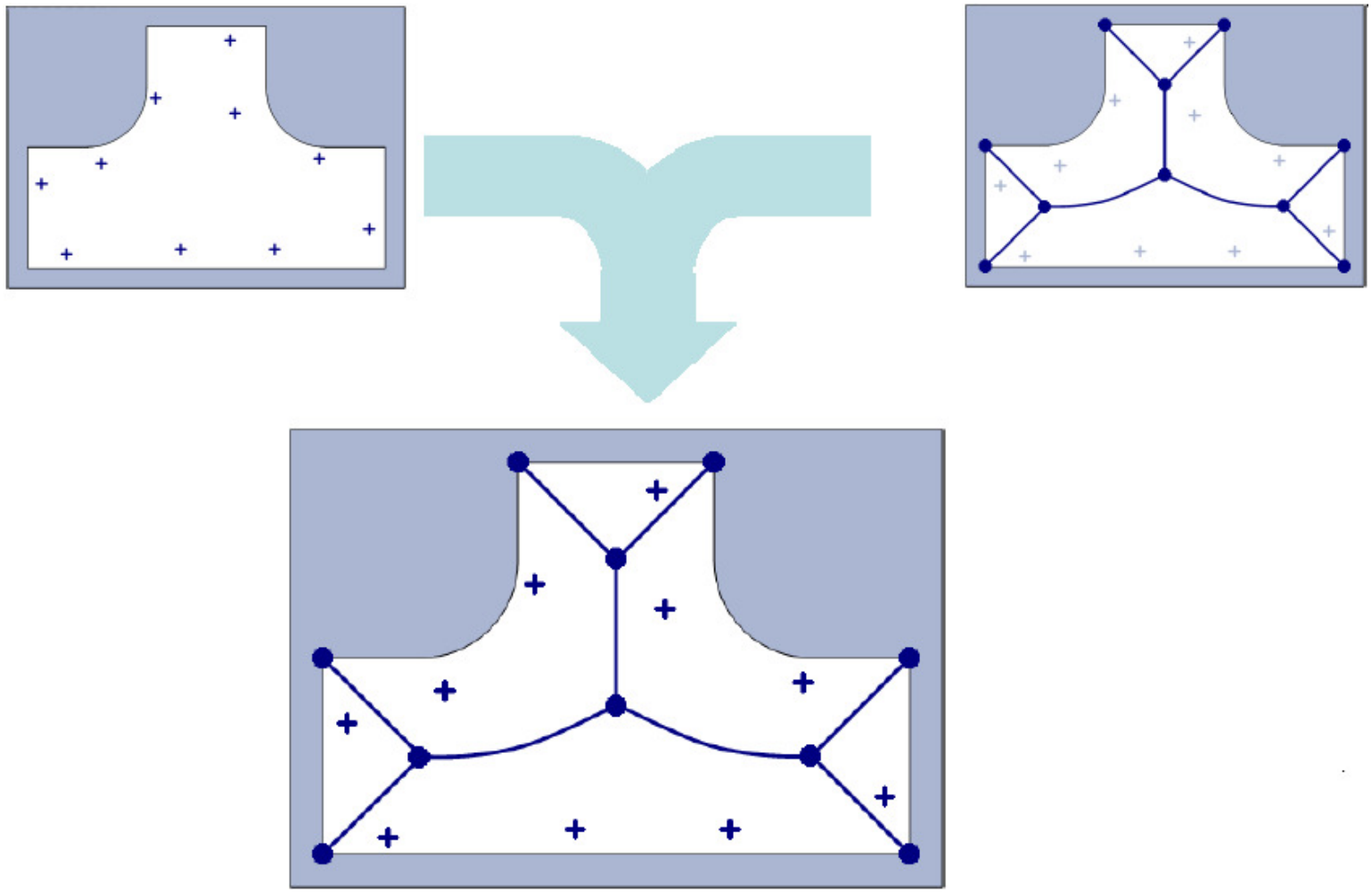
  → **There can not be "good" one-dimensional topological maps for $\Re^3$**



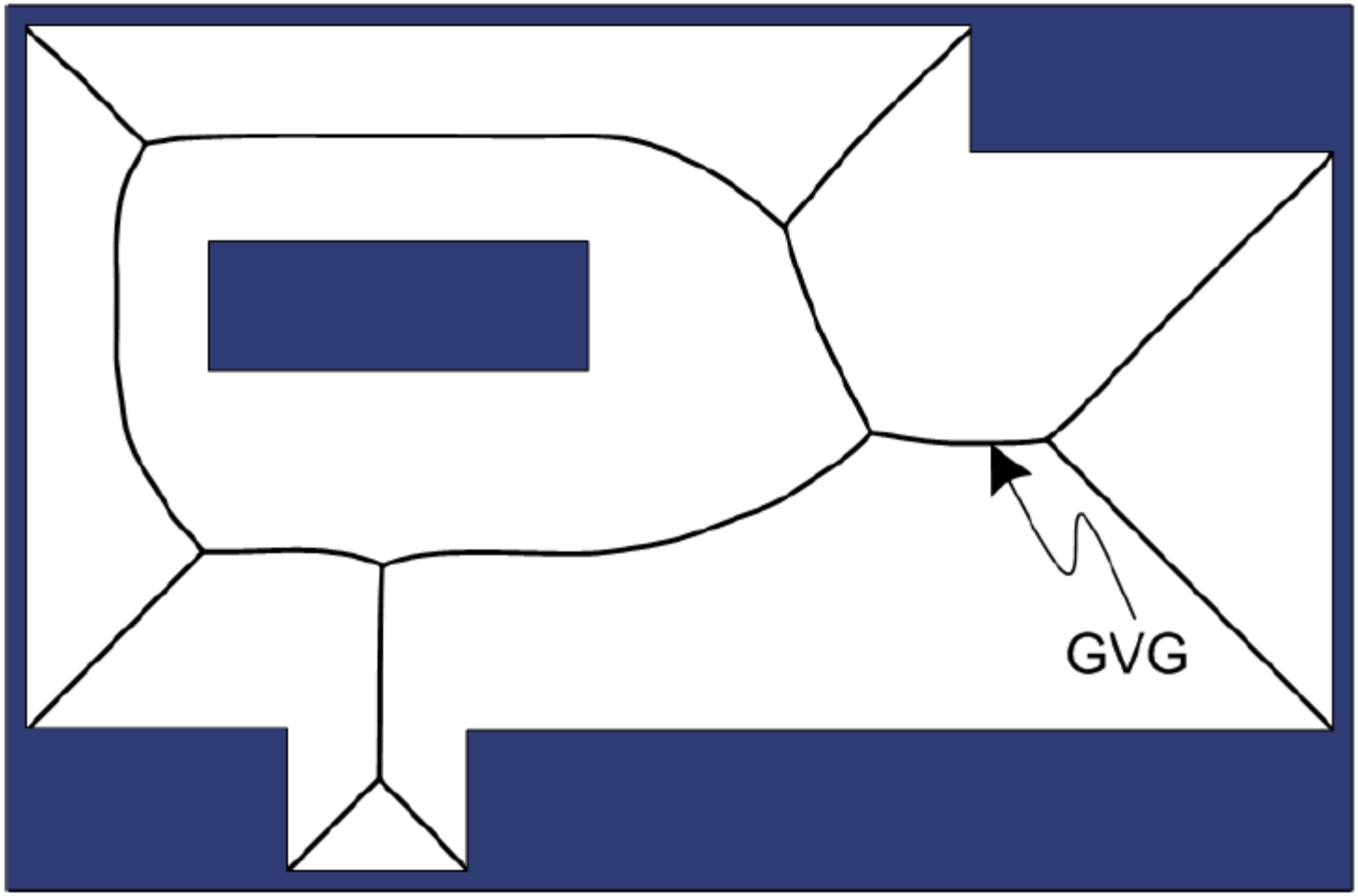


Application: Hierarchical SLAM

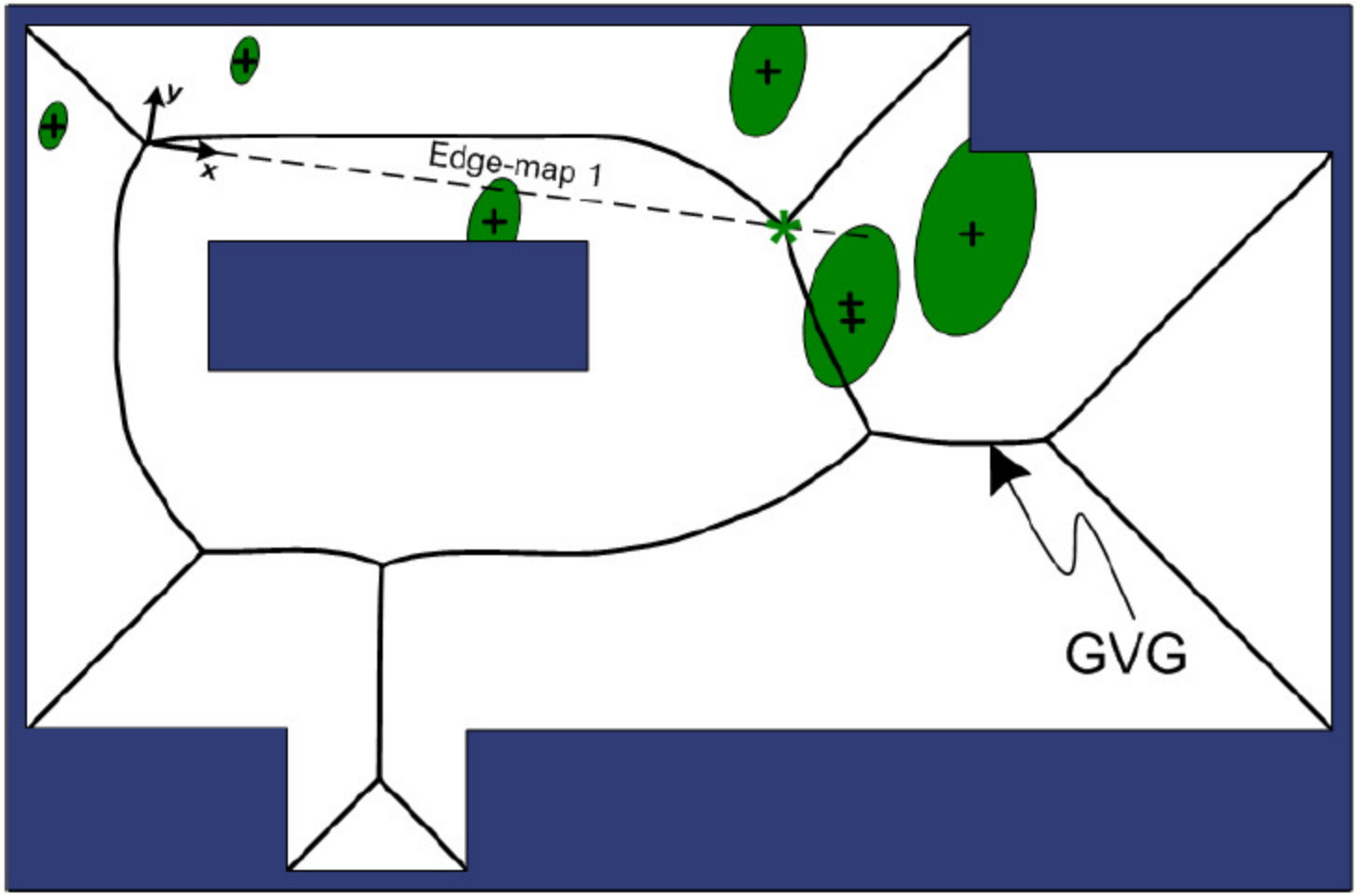
A feature-based technique in a topological framework



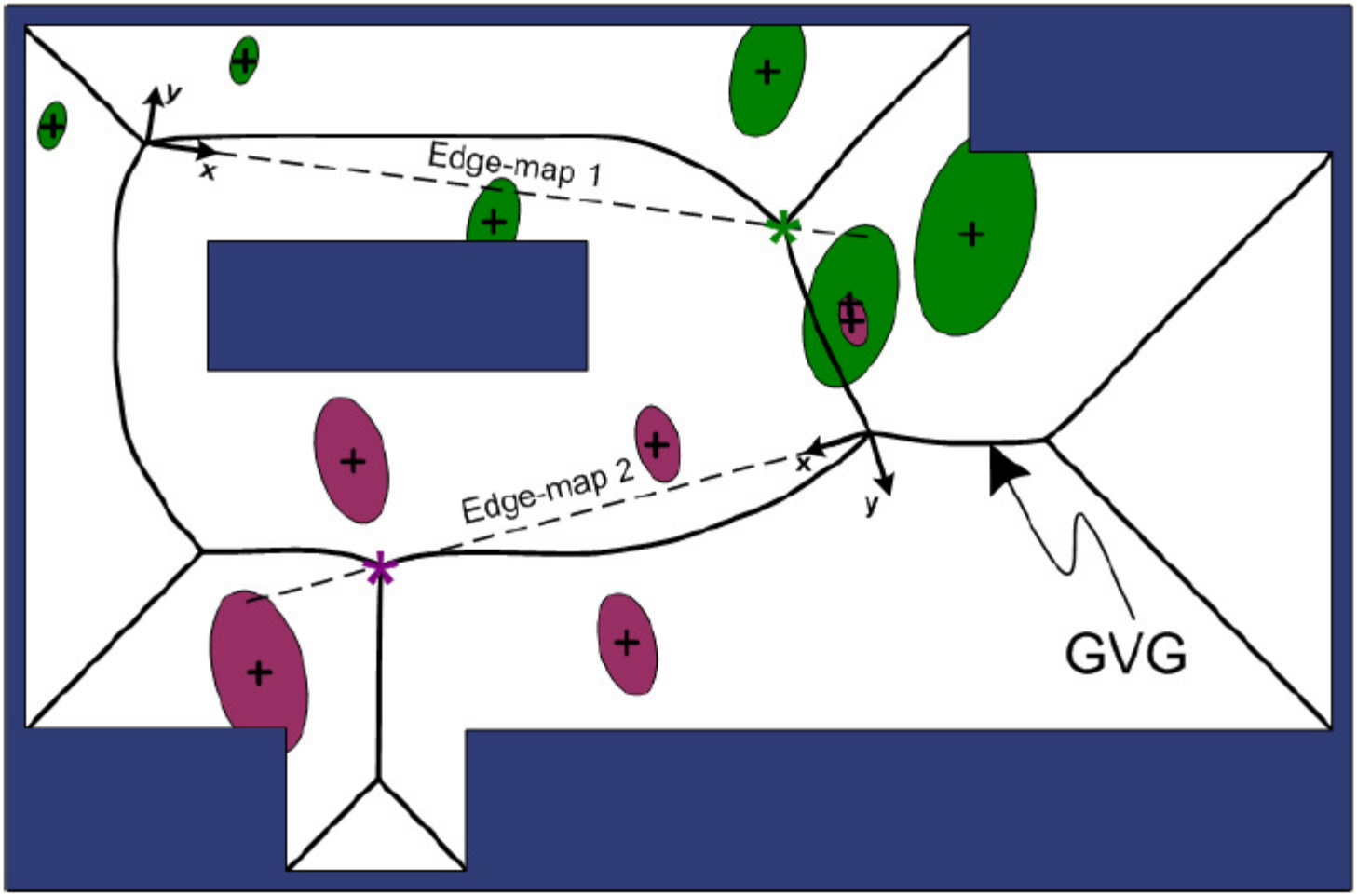
Embedded H-SLAM Map



Embedded H-SLAM Map



Embedded H-SLAM Map



Benefits&Drawbacks of Topological Maps

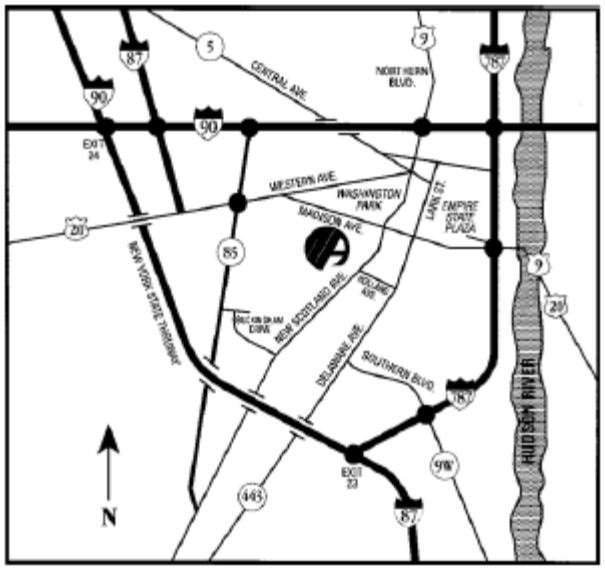
- Scale
  - dimension
  - geometric size
- Reduce planning problem
  - Graph search
  - Localization along a “line”
- Induces a hierarchy of maps for SLAM
  - T: Topological (Kuipers, Choset)
  - F: Feature-based (Leonard, Durrant-Whyte)
  - L: Local/Pixel-based (Morevac, Elfes, Thrun)
  - D: Dead-reckoning (Borenstein)
- Provides sensor space decomposition useful for control
  - Brooks and other: Behaviors – sense/act
  - Brockett; Manikonda, Krishnaprasad, and Hendler – Motion Description Languages
  - Rizzi, Burridge, Koditschek – Hybrid Controls
  - Kuipers and Choset – Topological Maps

- Cannot position in arbitrary locations

- Fails when environments is not topologically “rich”
  - –Hyper-symmetric
  - –Large open spaces

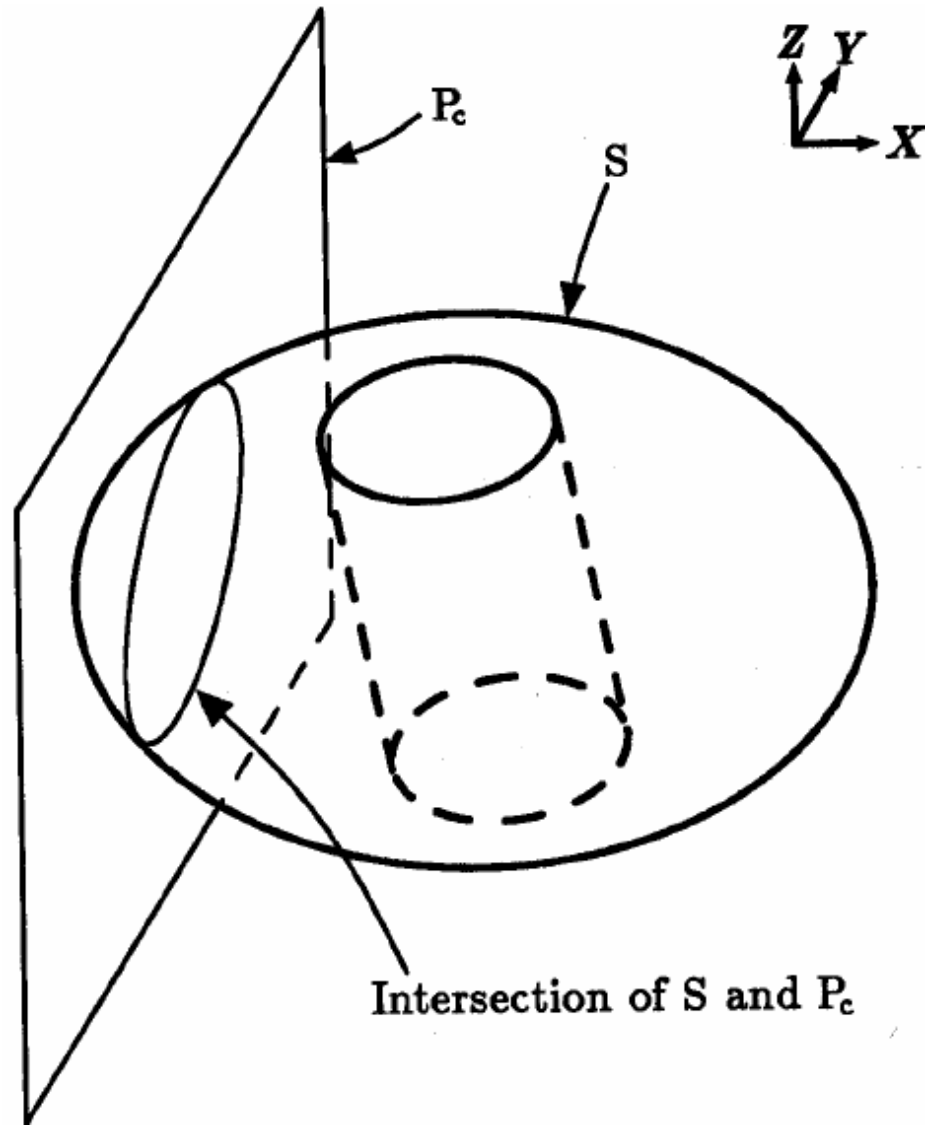
Silhouette Methods

- Canny's Roadmap Algorithm
- The Opportunistic Path Planner



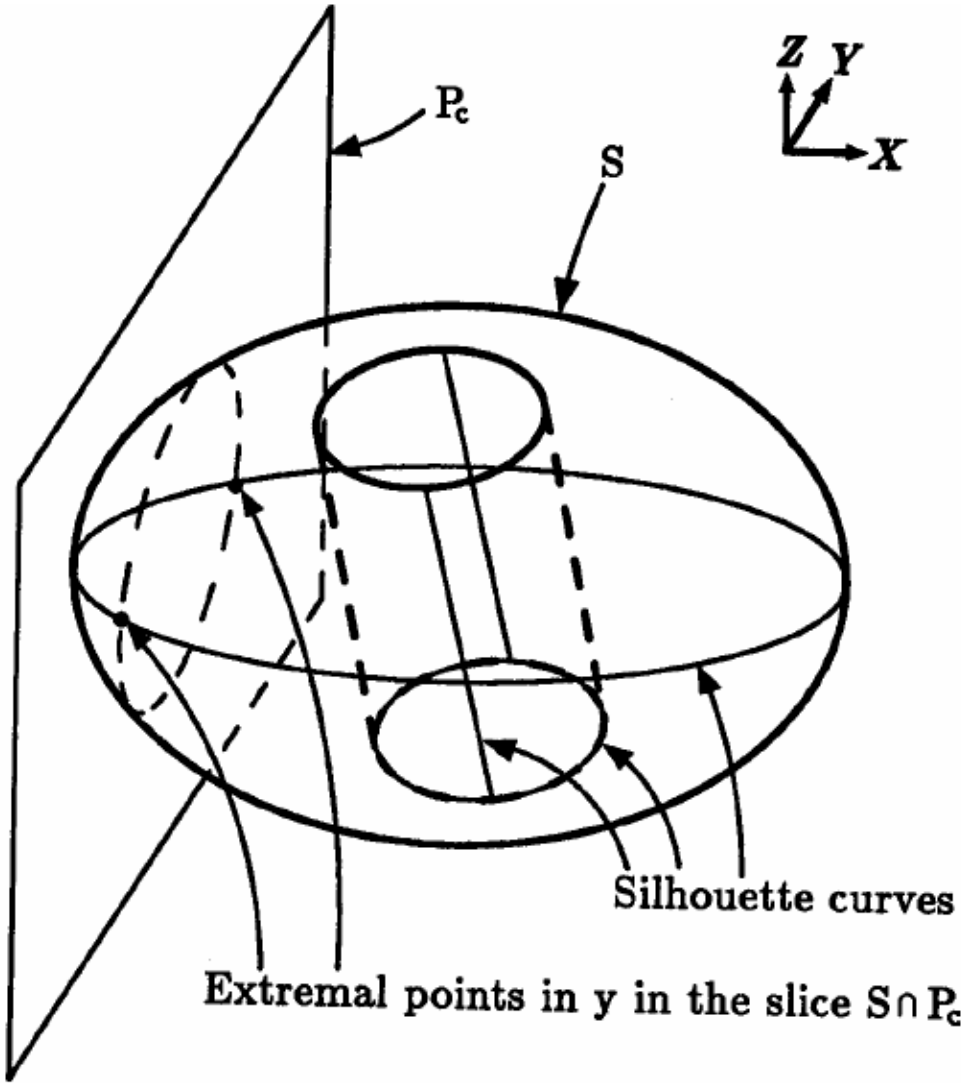
Illustrative Example (1)

- Let *S* be the ellipsoid with a through hole.
- P_c is a hyperplane of codimension 1 (x = c) which will be swept through *S* in the X direction.



Illustrative Example (2)

- At each point the slice travels along X we'll find the extrema in $S \cap P_c$ in the Y direction. If we trace these out we get **silhouette curves**.

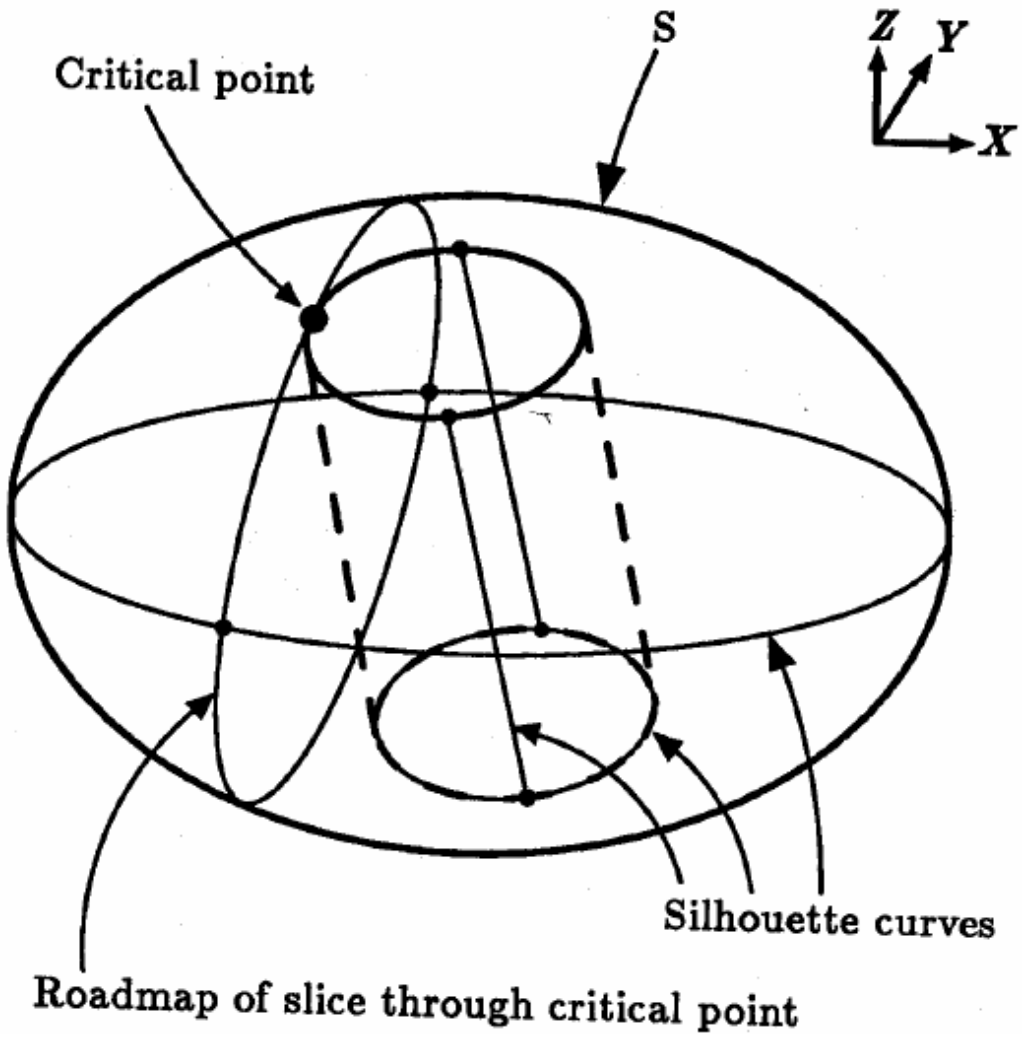


Illustrative Example (3)

- Observations:
  - The silhouette curves are one-dimensional.
  - This is not a roadmap, it's not connected.
  - There are points where extrema disappear and reappear, these will be called **critical points** and the slices that go through these points are **critical slices**.
  - A point on a silhouette curve is a critical point if the tangent to the curve at the point lies in P_c.

Illustrative Example (4)

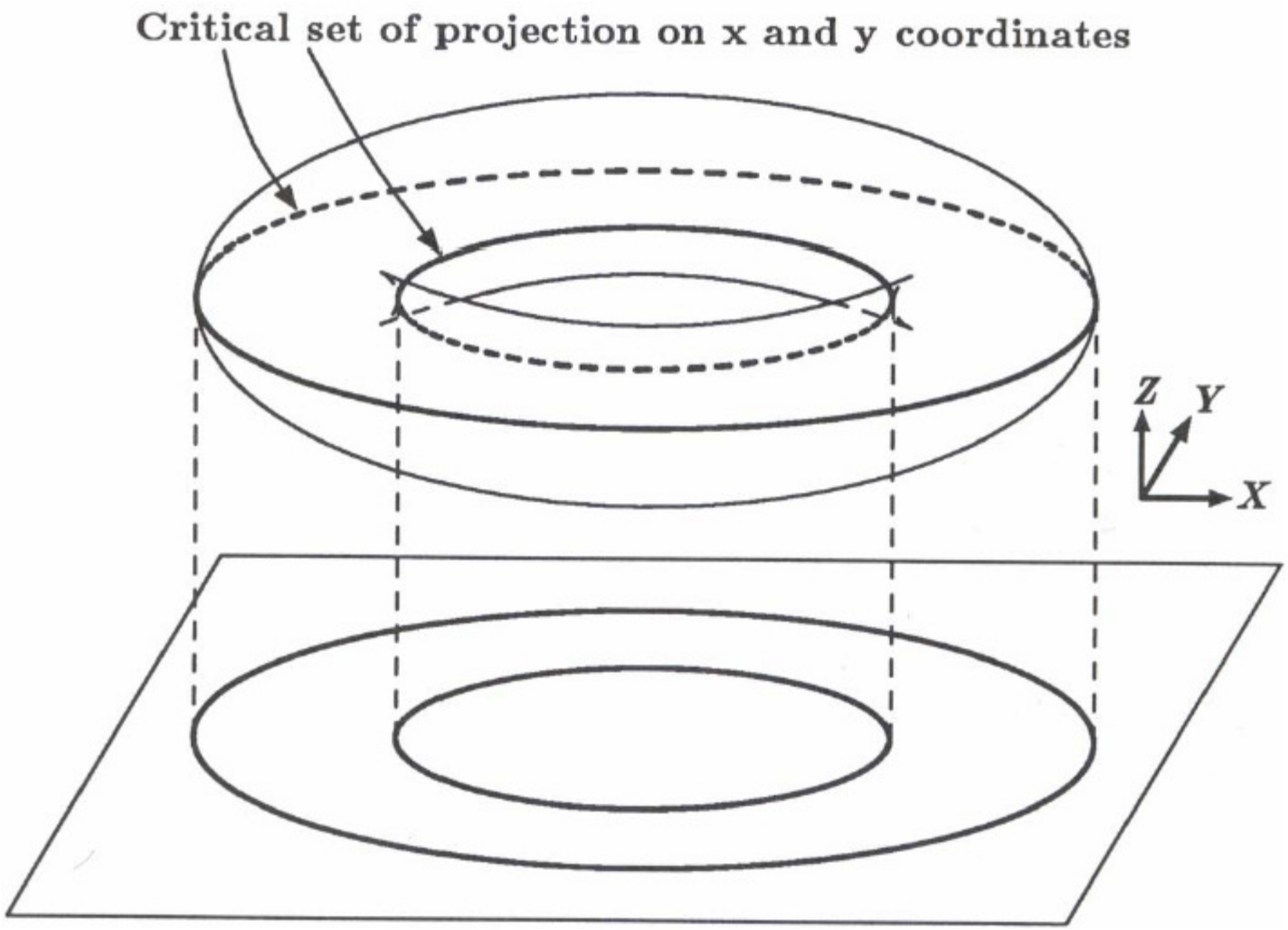
- We'll connect a critical point to the rest of the silhouette curve with a path that lies within $S \cap P_c$. This can be done by running the algorithm recursively. Each time, we increase the codimension of the hyperplane by 1.



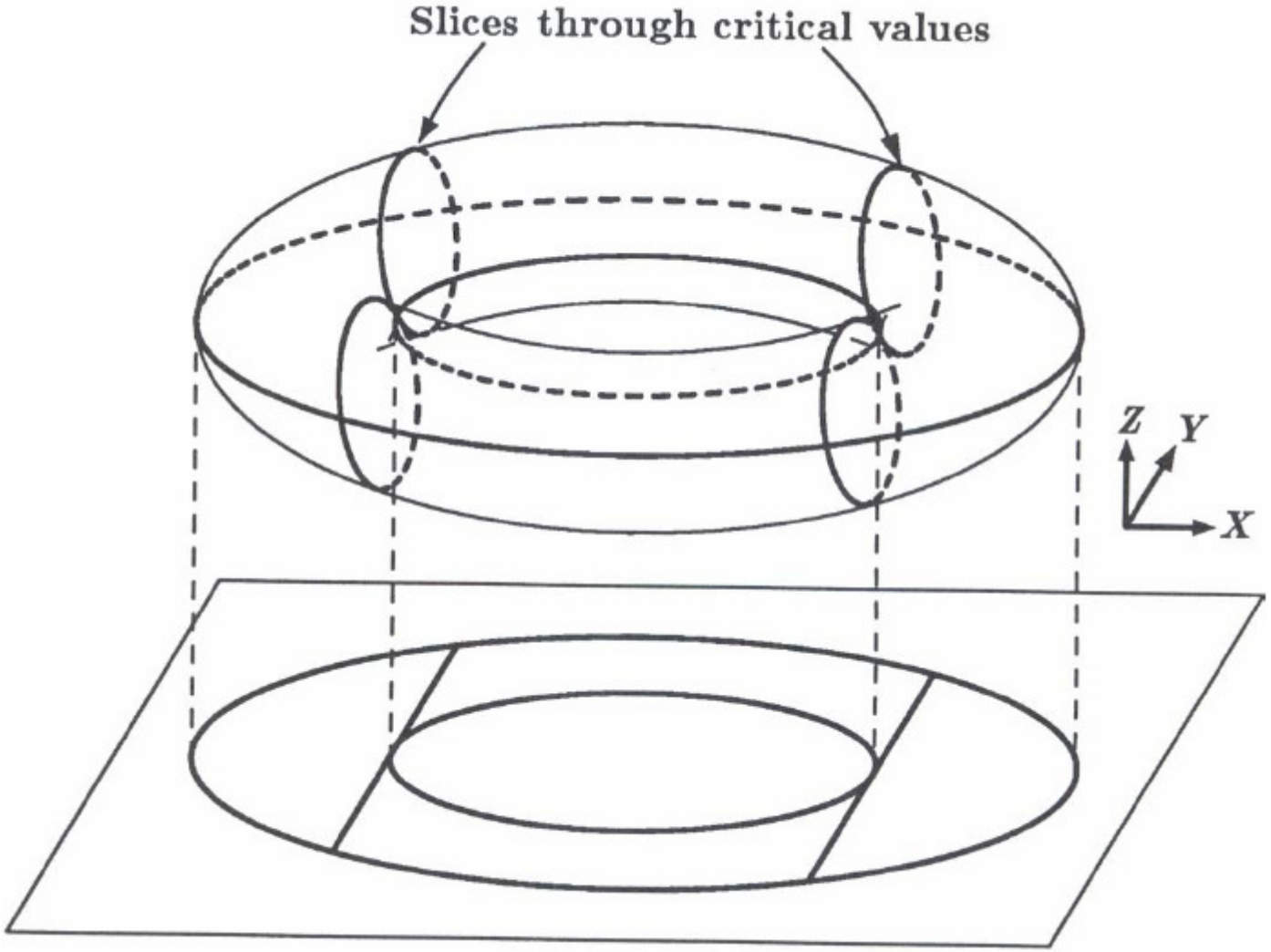
Illustrative Example (5)

- Final points
  - The recursion is repeated until there are no more critical points or the critical slice has dimension 1(it is its own roadmap)
  - The roadmap is the union of all silhouette curves

Another Example (1)



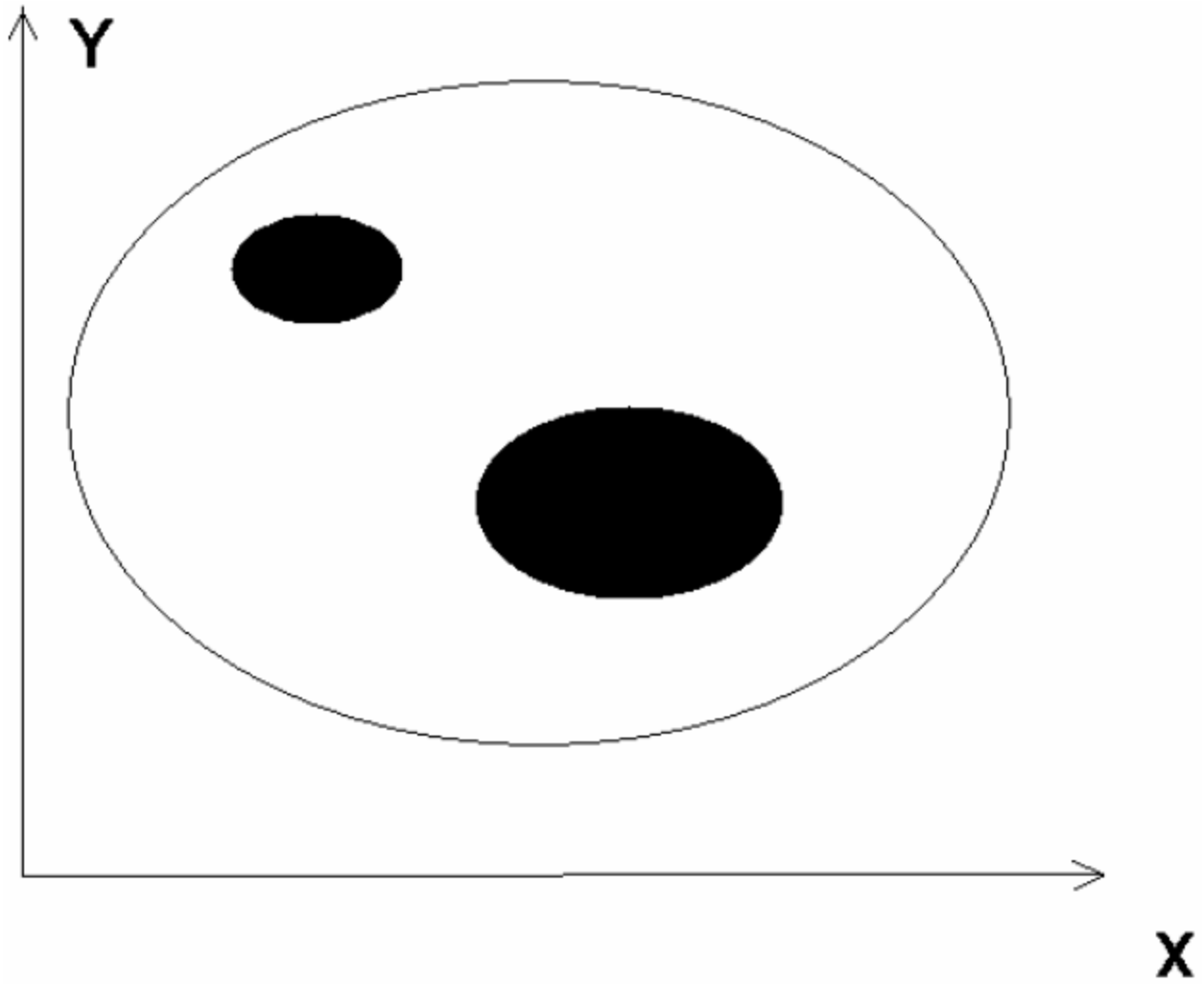
Another Example (2)



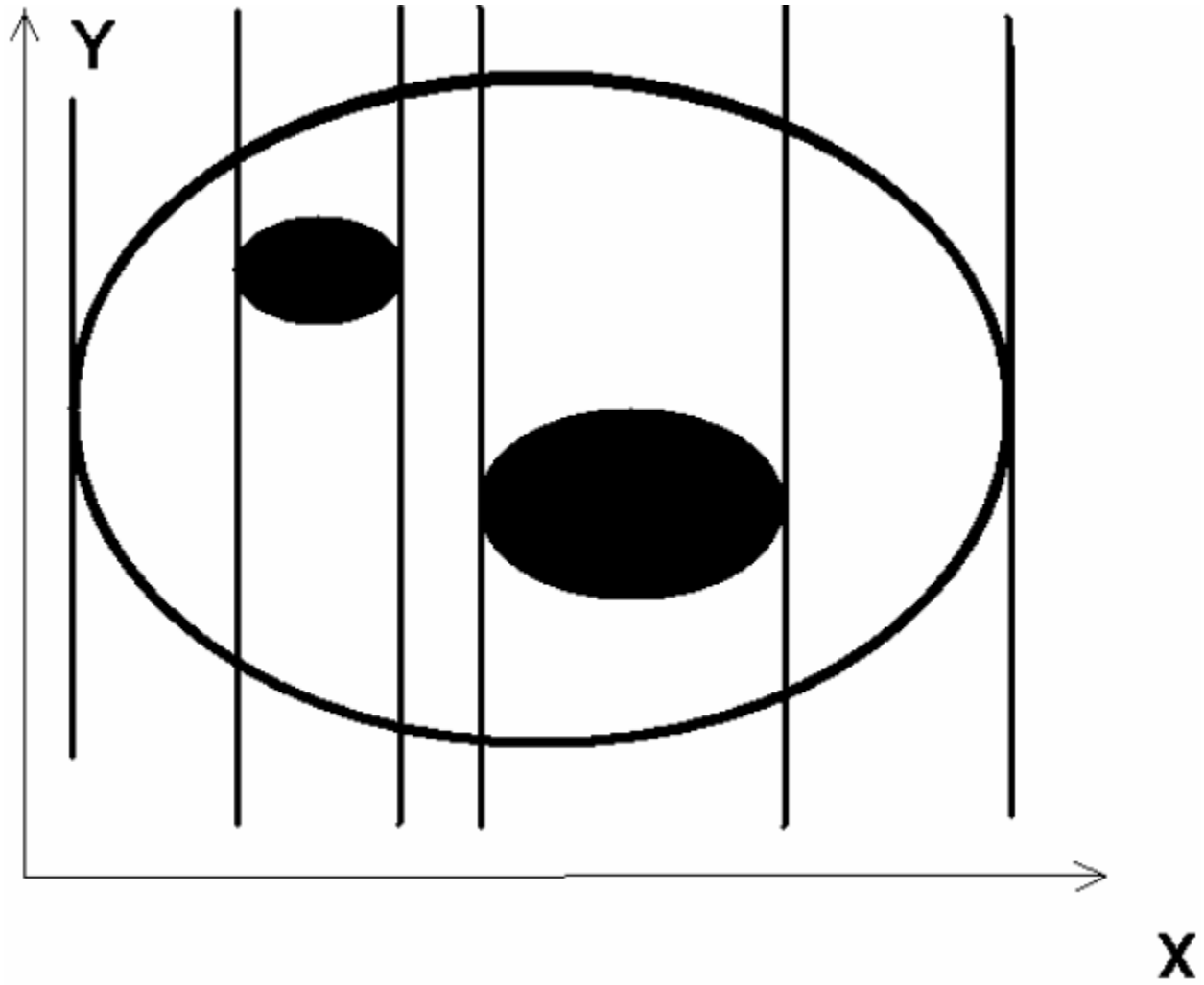
Accessibility and Departability (1)

- In order to access and depart the roadmap we treat the slices which contain q_s and q_g as critical slices and run the algorithm the same way.

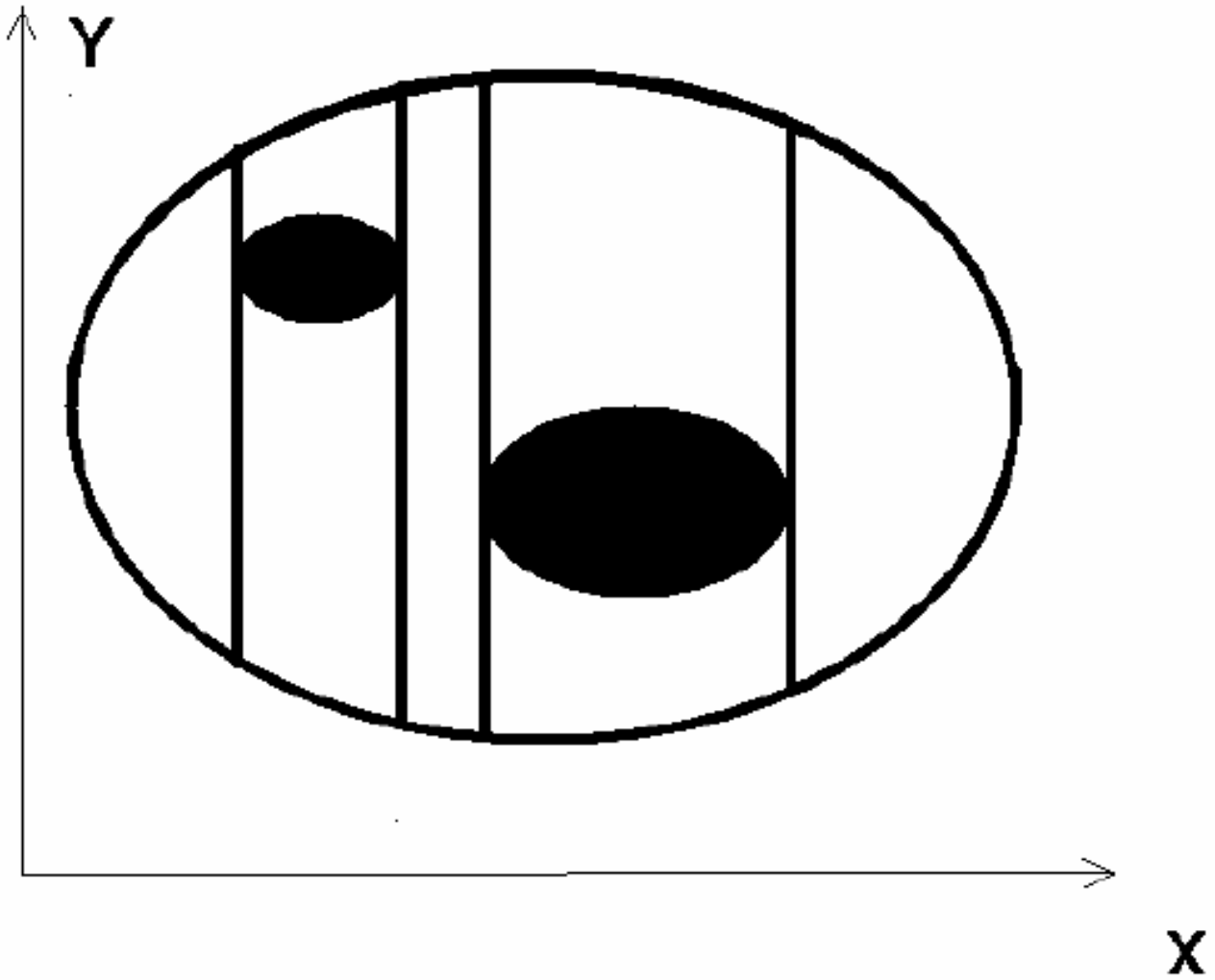
Accessibility and Departability (2)



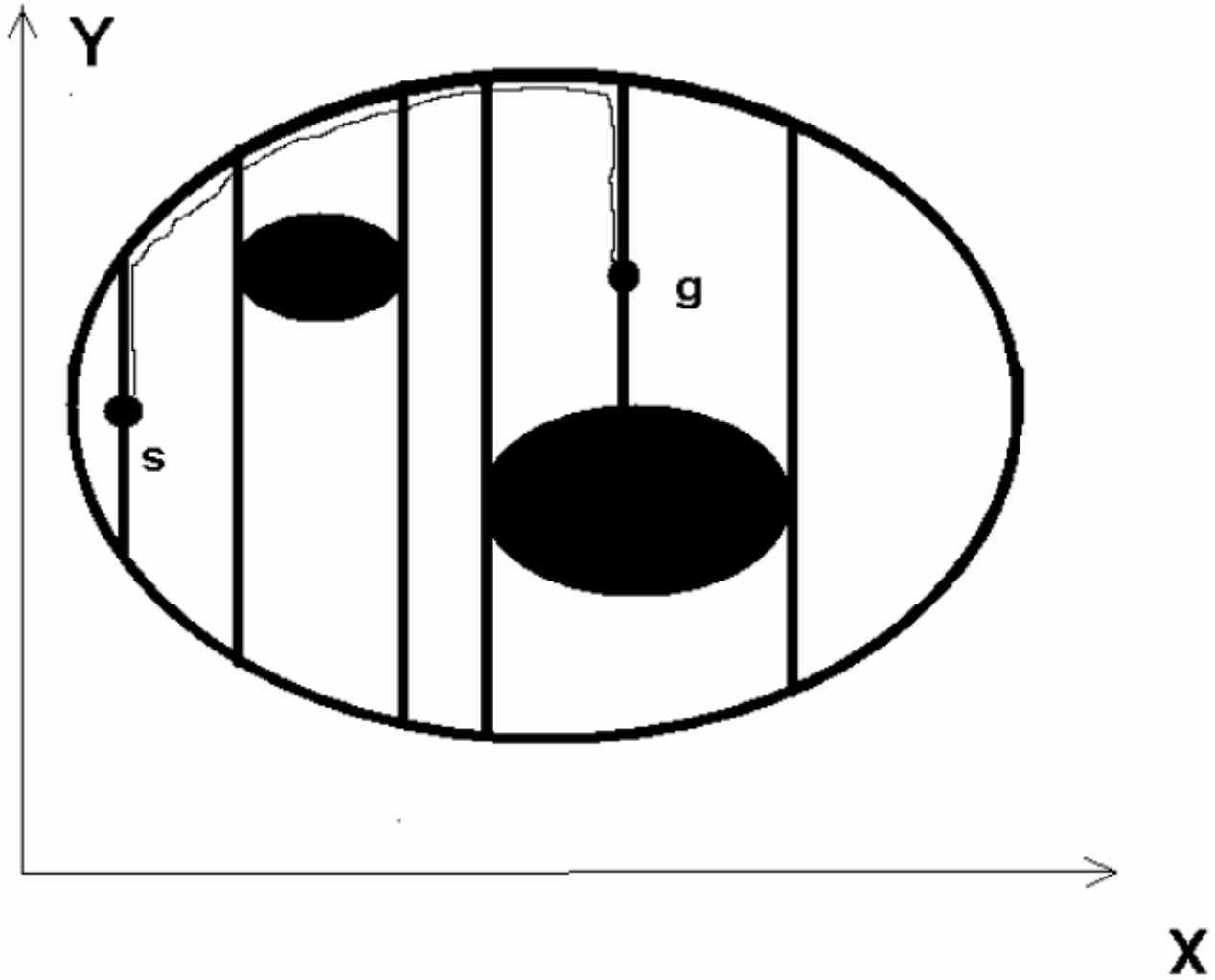
Accessibility and Departability (3)



Accessibility and Departability (4)



Accessibility and Departability (5)



Building the Roadmap

- Given that the algorithm is now clear conceptually, let's establish the mathematical machinery to actually construct the roadmap. We must define
  - The sets
  - The slices
  - How to find extrema
  - How to find critical points

The Sets

- The S which this algorithm deals with are ***semi-algebraic sets*** that are closed and compact.
  - **Def**: A *semi-algebraic set* $S<\Re^r$ defined by the polynomials $F_1,..,F_n$ in Q_r is a set derived from the sets
    $S_i=\{x \text{ in } \Re^r \mid F_i(x)>0\}$
    by finite union, intersection and complement.
  - Ex: $(x^2+y^2 \leq 1)$ and $(z \leq 1)$ and $(z \geq -1)$

The Slices

- The slices are the intersection of a hyperplane and S

$$S_c = S \cap P_c = \{x \in S : \pi_1 = c\}$$

$$\bigcup_c S_c = S$$

- where π_1 is the projection on to the first coordinate

$$\pi_k(x_1, x_2, \ldots, x_n) = x_k$$

How To Find Extrema

- When constructing the silhouette curves, we look for extrema of $\pi_2|S_c$, the extrema of the projection of S_c in a second direction.

- In order to find the extrema on a manifold we will refer to the **Lagrange Multiplier Theorem**.

How To Find Extrema

- **Lagrange Multiplier Theorem**:

  Let S be an n-surface in $\Re^{n+1}$, $S = f^{-1}(c)$ where $f: U \rightarrow \Re$ is such that $\text{Grad}(f)(q) \neq 0$ for all q in S.

  Suppose $h: U \rightarrow \Re$ is a smooth function and p in S is an extremum point of h on S. Then for all λ in $\Re$ s.t. $\text{Grad}(h)(p) = \lambda \text{Grad}(f)(p)$ (they are parallel)

How To Find Extrema

- Example:
  - Consider $S=f^{-1}(0)$ where $f=x^2+y^2+z^2-1$ (a solid unit sphere). Extrema of $h=\pi_1(x,y,z)=(x)$.

$$d(f,h)=\begin{bmatrix} 2x & 2y & 2z \\ 1 & 0 & 0 \\ & & \end{bmatrix}$$

y = z = 0 (y-z plane) and only points on sphere is x = 1, x = -1, left most and right most

How To Find Extrema

Canny's Generalization of the Lagrange Multiplier Theorem:

Suppose that *U* is an open subset of the kernel of some map f:$\Re^r \rightarrow \Re^n$, and let *f* be transversal to {0}. Let *g*:$\Re^r \rightarrow \Re^m$ be a map, then *x* in *U* is an extremum of *g* | *U* iff the following matrix is not full rank.

$$d(f,g)_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x) & \cdots & \frac{\partial f_1}{\partial xr}(x) \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1}(x) & & \frac{\partial f_n}{\partial x_r}(x) \\ \frac{\partial g_1}{\partial x_1}(x) & \cdots & \frac{\partial g_1}{\partial x_r}(x) \\ \vdots & & \vdots \\ \frac{\partial g_m}{\partial x_1}(x) & \cdots & \frac{\partial g_m}{\partial x_r}(x) \end{bmatrix}$$

How To Find Extrema

Canny's Slice Lemma:

The set of critical points of $\pi_{12}|S$, $\Sigma(\pi_{12}|S)$, is the union of the critical points of $\pi_2|S_c$ where we sweep in the 1 direction.

$$\Sigma(\pi_{12}|_S) = \bigcup_{\lambda} \Sigma(\pi_2|_{\pi_1^{-1}(\lambda)}).$$

How To Find Extrema

Example:

Consider $S=f^{-1}(0)$ where $f=x^2+y^2+z^2-1$ (a solid unit sphere). If we sweep in the x direction and extremize in the y direction $h=\pi_{12}(x,y,z)=(x,y)$.

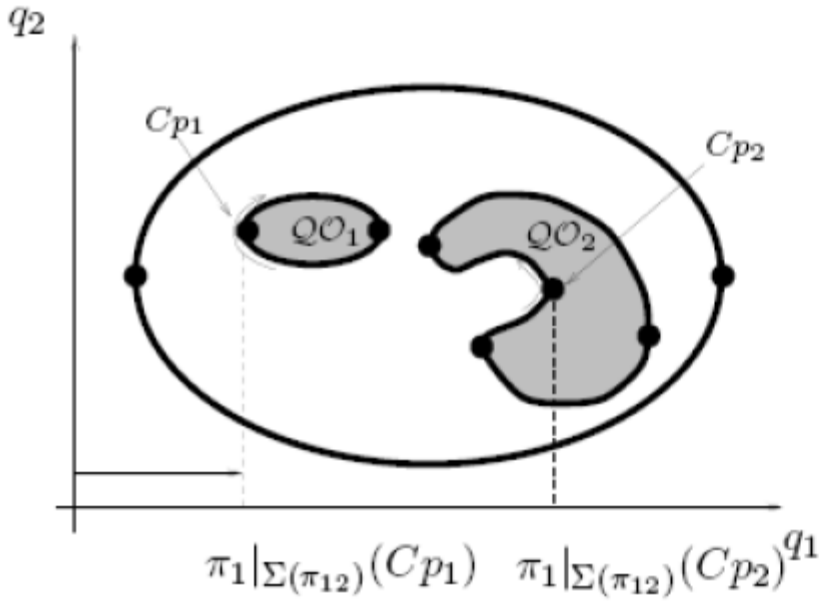
$$d(f,h)=\begin{bmatrix} 2x & 2y & 2z \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

So the silhouette curve is the unit circle on the x-y plane

Finding Critical Points

The critical points which denote changes in connectivity of the silhouette curves also follow from Canny's Generalization. They are the extrema of the projection on to the sweeping direction of the silhouette curves. Simply

$$\Sigma(\pi_{1|\Sigma(\pi_{12})})$$

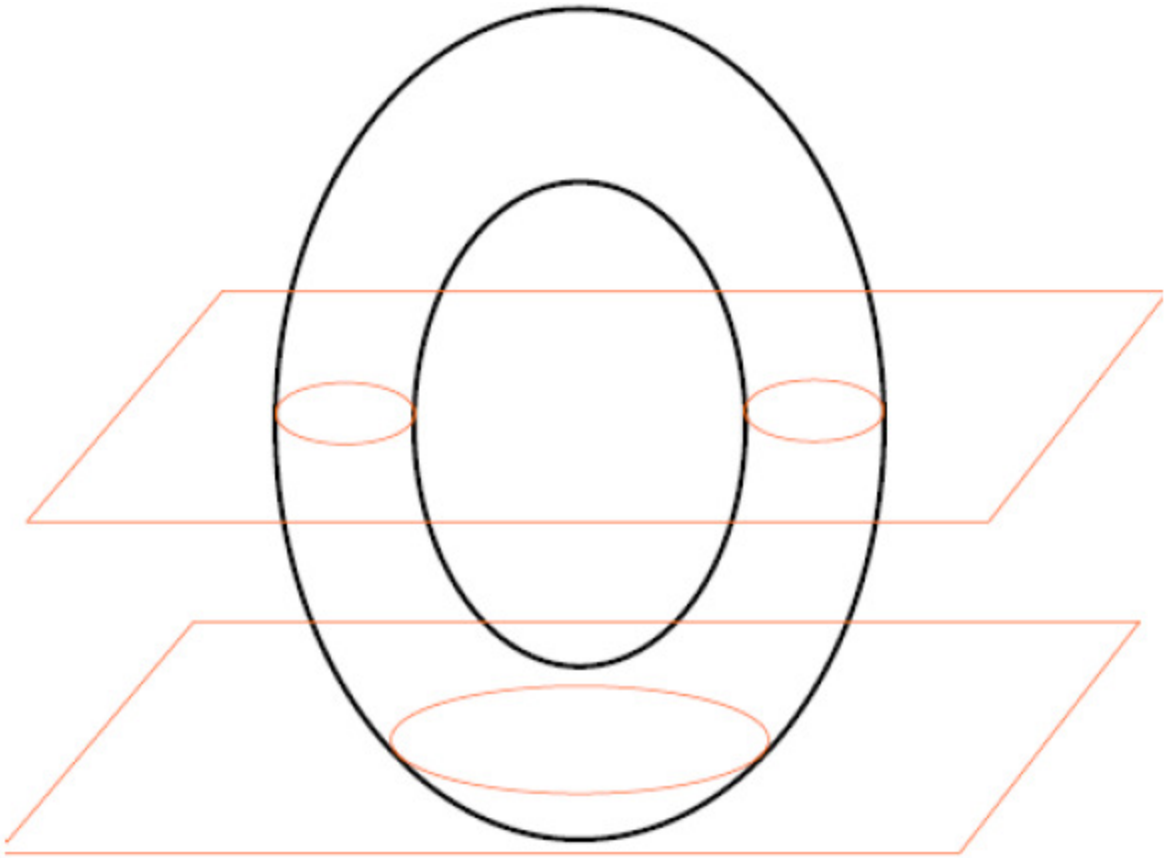


$\pi_1(q)$

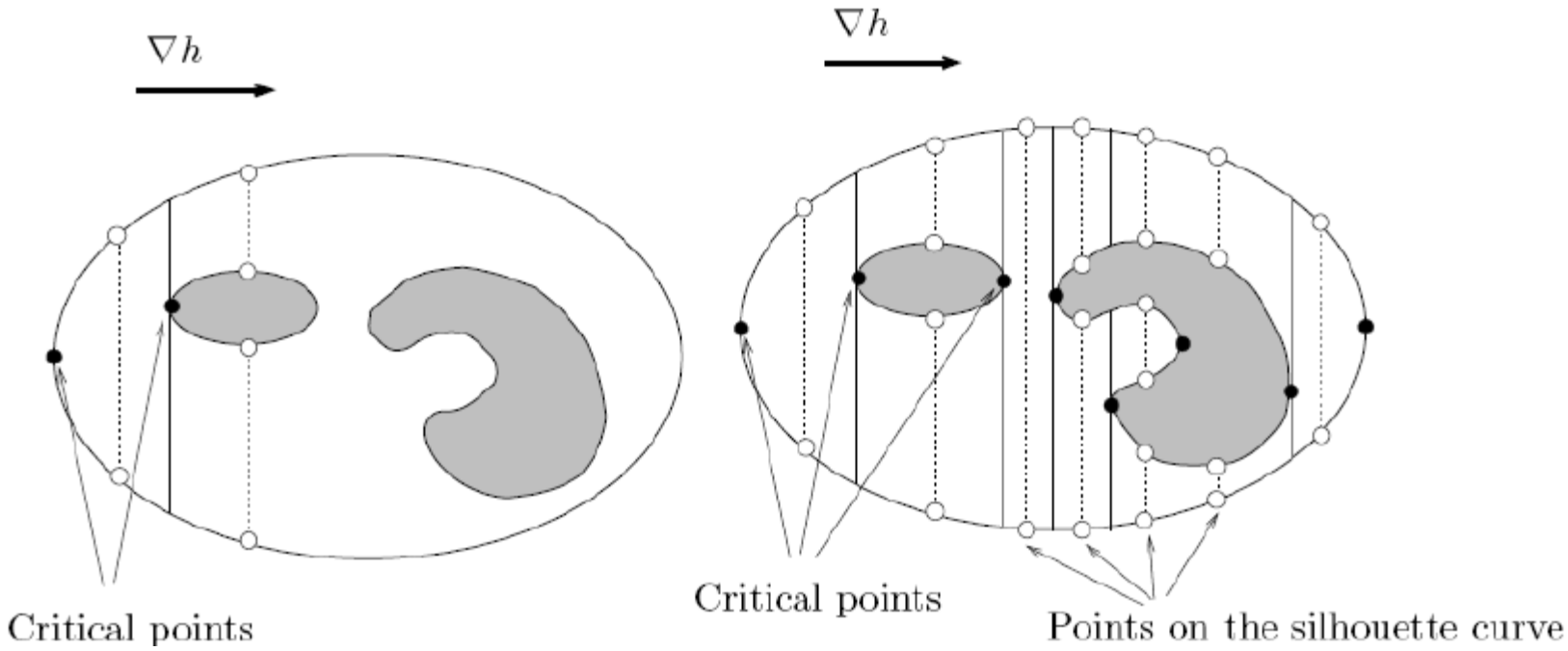
Can be viewed as the distance to the y axis from a point

Critical point is where roadmap tangent is parallel to slice

Connectivity change at Critical Points

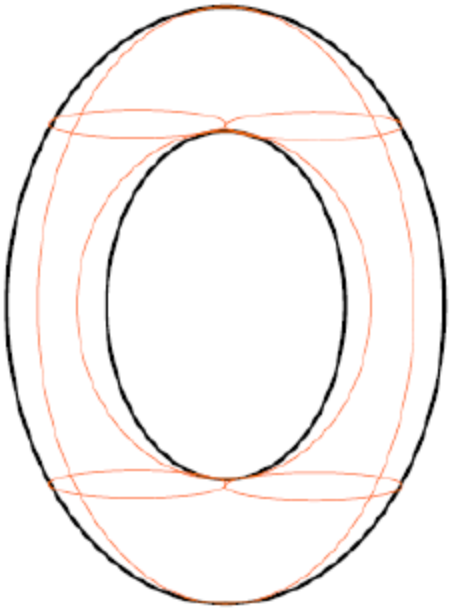


Between Critical Points



Building the Roadmap (Conclusion)

- We can now find the extrema necessary to build the silhouette curves
- We can find the critical points where linking is necessary
- We can run the algorithm recursively to construct the whole roadmap



The Opportunistic Path Planner

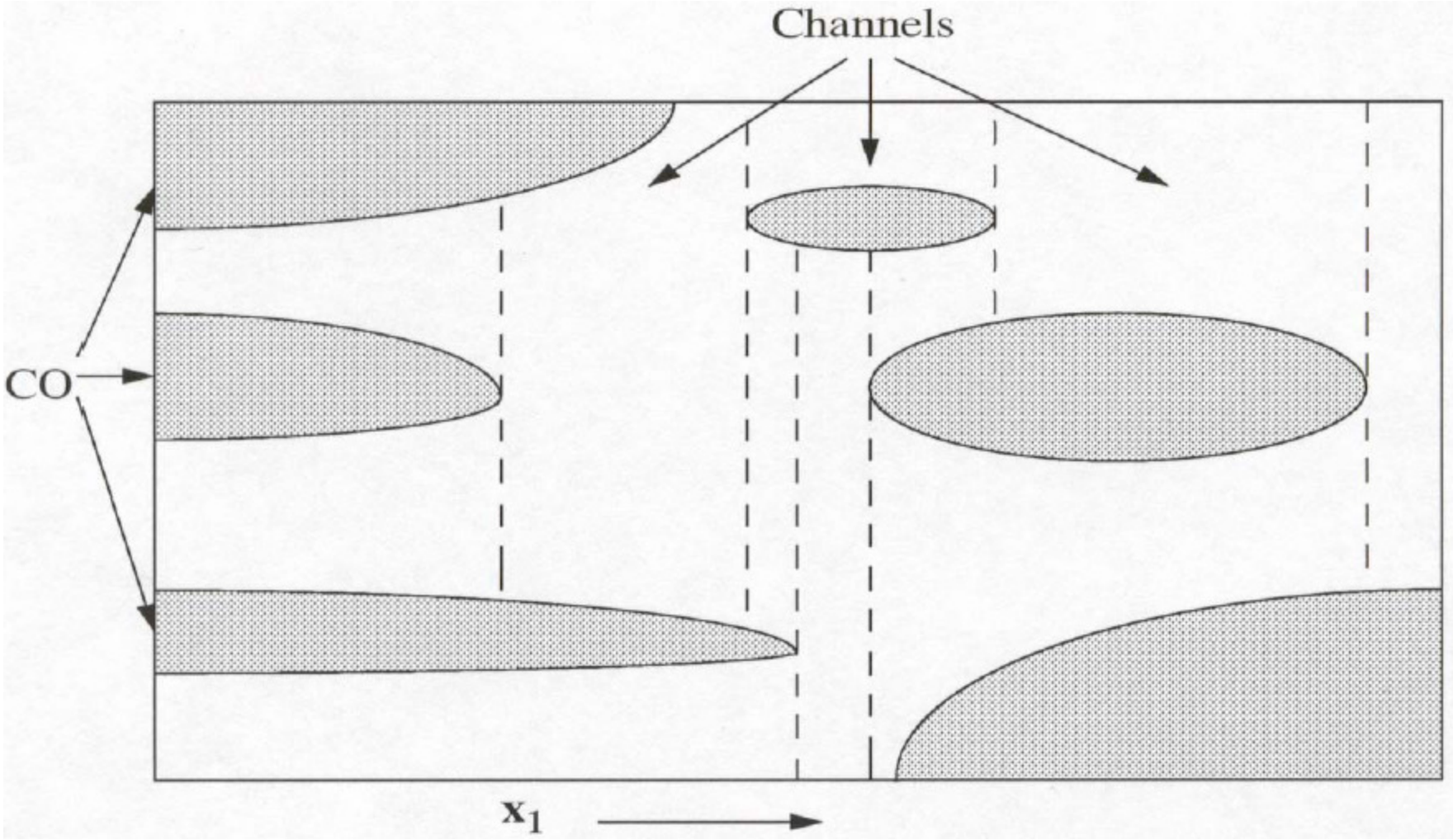
- The Opportunistic Path Planner is similar to Canny's Roadmap but differs in the following ways
  - Silhouette curves are now called *freeways* and are constructed slightly differently
  - Linking curves are now called *bridges*
  - It does not always construct the whole roadmap
  - The algorithm is not recursive

Channels

Def: A *channel slice* is a slice at which the connectivity of the intersection with the sweeping hyperplane and the freespace changes.

Def: A *channel* is a subset of the freespace which is bounded by channel slices and configuration space obstacles

Channels



Freeways

- Freeways are defined by the following artificial potential field which induces an artificial repulsion from the surface of obstacles

$$U_{art}(x) = D(x)$$

- A freeway is the locus of the maxima of $U_{art}(x)$ as you sweep through the configuration space

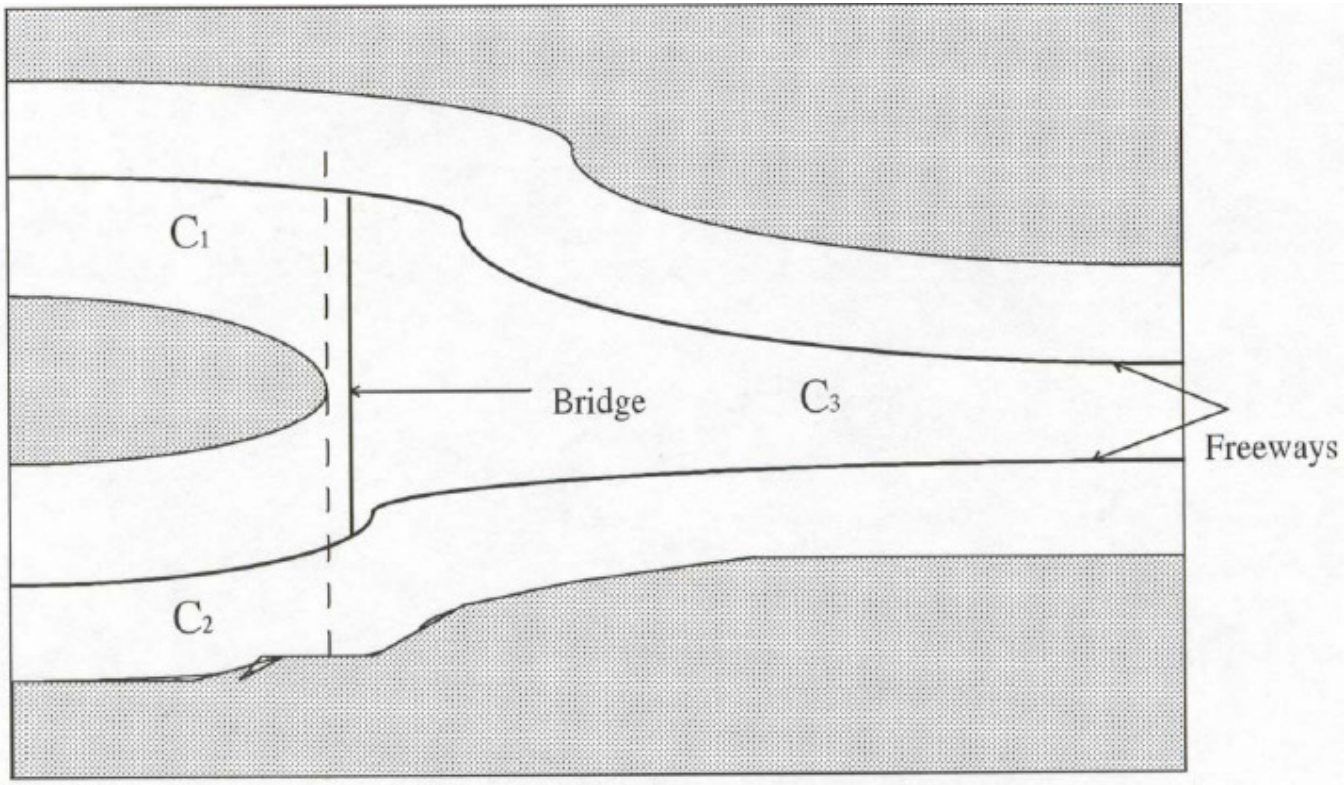
Interesting Critical Points and Inflection Points

Def: An *interesting critical point* is a critical point that corresponds to the joining or splitting of the intersection of the sweeping hyperplane and the freespace

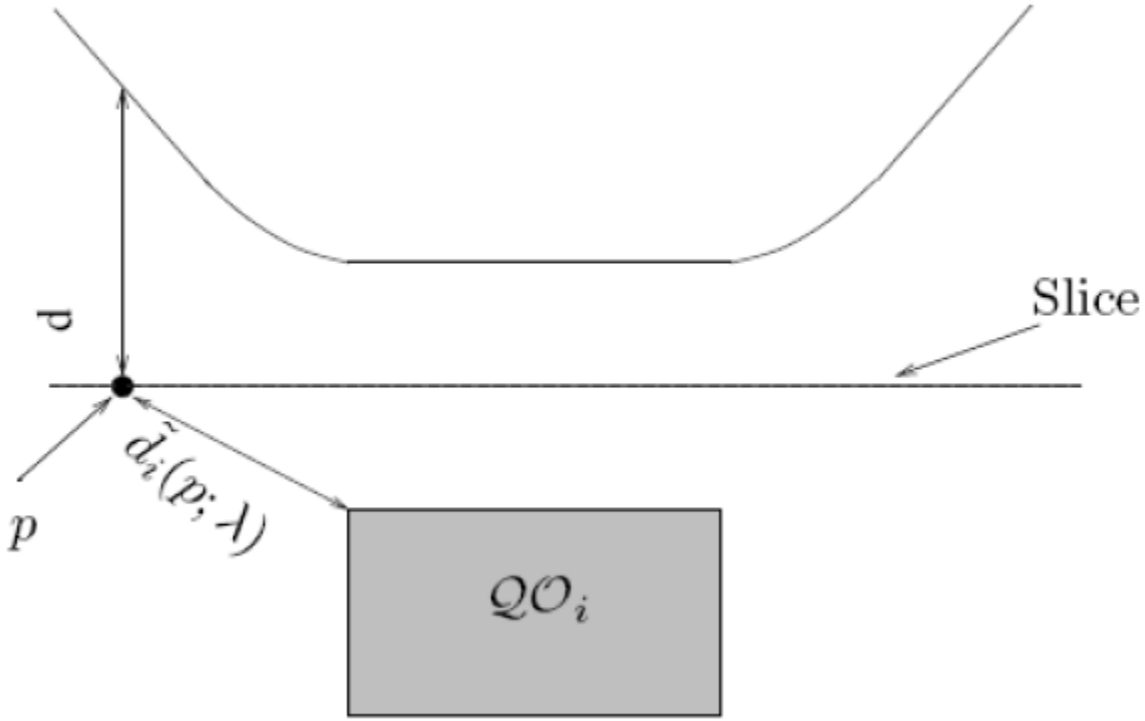
Def: An *inflection point* is a point where the tangent to the freeway curve becomes orthogonal to the sweep direction

Bridges

- **Def:** A *bridge* is a one-dimensional set which links freeways from channels that have just joined or are about to split (as you sweep across)



Freeway Tracing



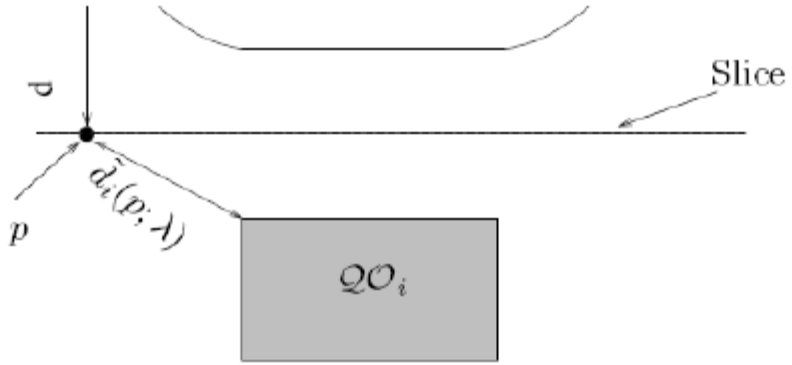
Freeway Tracing

- Freeway tracing is done by tracking the locus of the maxima of the artificial potential field and terminates when

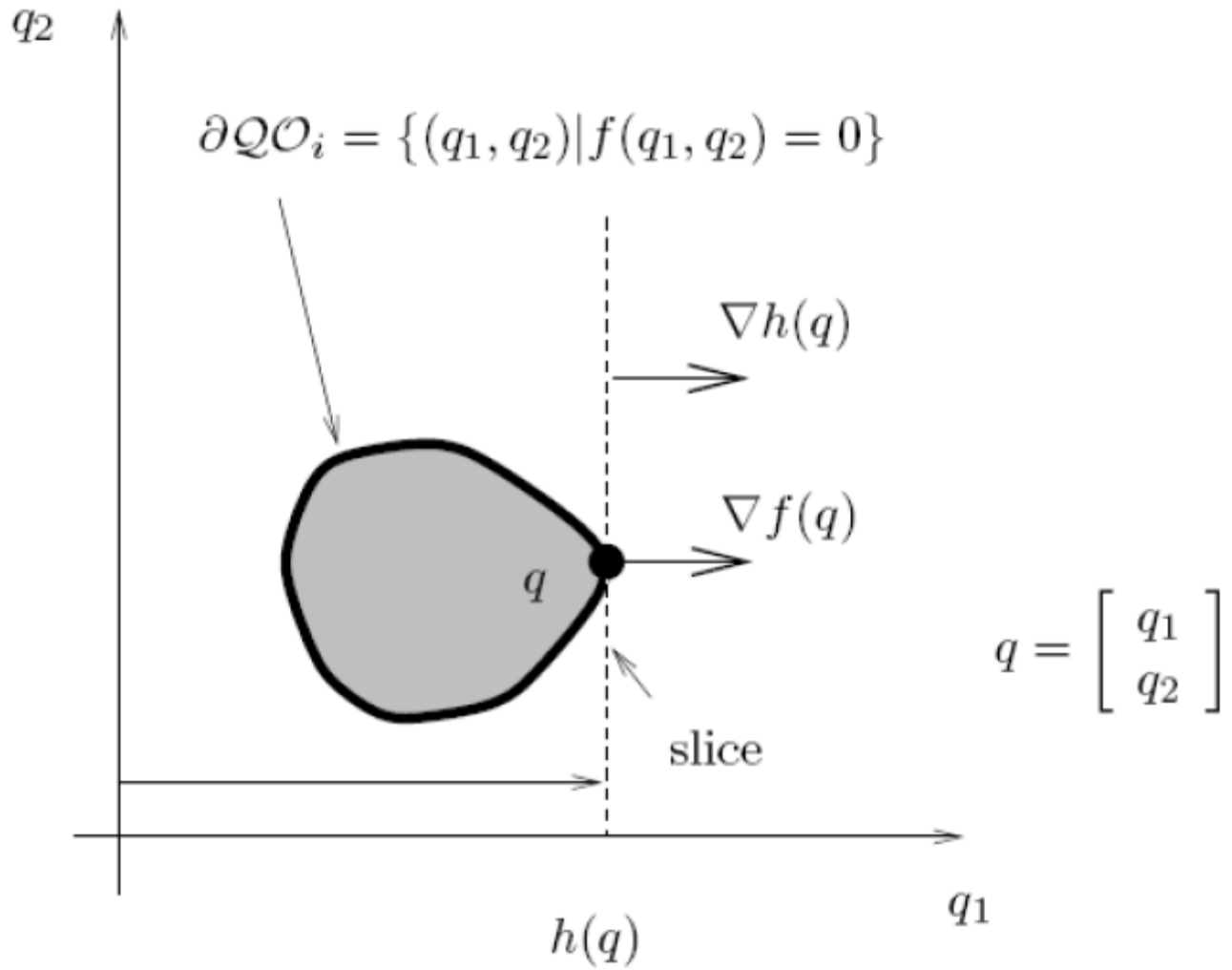
  (1) The freeway runs into an inflection point where you create a bridge

  (2) The freeway runs into an obstacle where it ends

$$\partial D(q^*) = \mathrm{Co}\{\nabla d_i(q^*) \mid i \in Z(q^*)\}$$
$$= \sum_{i \in Z(q^*)} \mu_i \nabla d_i(q^*) \text{ where } \sum_{i \in Z(q^*)} \mu_i = 1 \text{ and } \mu_i > 0,$$



Also create bridges at interesting critical points



Accessibility and Departability

- The roadmap is accessed and departed by connecting q_s and q_g to a local maximum on the slice which they reside (which is part of a freeway).

- This is referred to as *hill-climbing* and is the same procedure we use when creating bridges except in the case of bridges we hill-climb in two directions.

Building the Roadmap

(1) Hill-climb from both q_s and q_g. Then trace freeway curves from both start and goal

(2) If the curves leading from start and goal are not connected enumerate a split point or join point and add a bridge curve near the point. Else stop.

(3) Find all points on the bridge curve that lie on other freeways and trace from these freeways. Go to step 2.

Summary

- Roadmap methods create a graph of “roads” that will move you through the space; just get on and get off again

- The visibility graph is one method of doing this for polygonal worlds
- Voronoi diagrams are a second form of roadmap

- We will see more graphs in the second half of the semester...