CENG 222

Statistical Methods for Computer Engineering

Week 14

Chapter 6 Stochastic Processes

Counting Processes

Simulation of Stochastic Processes

Counting Processes

- X is a counting process if $X(t)$ shows the number of items counted by time $t \in T$.
- Counting processes are non-decreasing
- Since they show count, they are discrete-state processes
- Can be *discrete-time* (Binomial Process) or *continuous-time* (Poisson Process)
- Examples:
  - Counting emails received by time t
  - Counting total number of goals scored in a game by time t

Binomial process

- Discrete-time (i.e., each time step contains a Bernoulli trial)
- A binomial process $X(t)$ is the number of successes by the time t in a sequence of independent Bernoulli trials.
- $X(t)$ number of successes by the time t
  - Binomial(tp)
- Y number of trials between two successes
  - *Geometric(p)*

Binomial Process

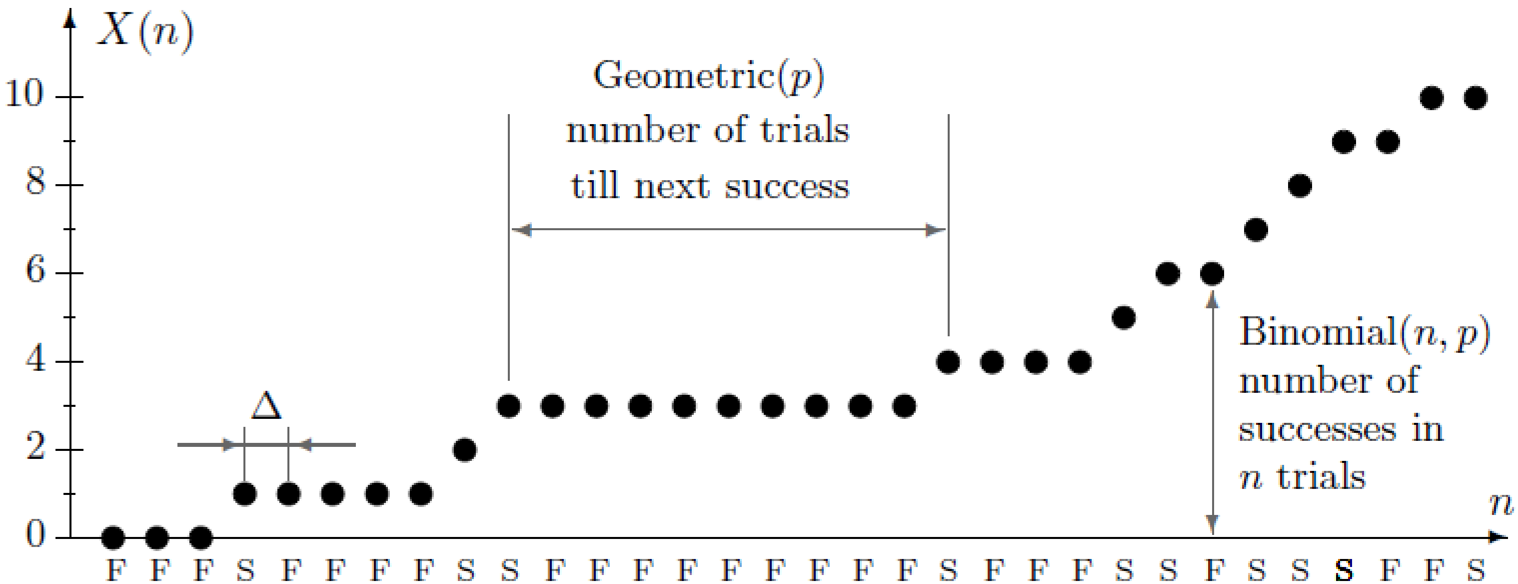

FIGURE 6.7: *Binomial process (sample path). Legend:* S = *success*, F=*failure.*

of trials versus time

- Although discrete, if the time unit for each trial is not 1 (second, minute, etc.), we may need to be careful in using the value of t in our computations.
- For example: if a Bernoulli trial occurs every 3 seconds, $X(6)$ is **not** Binomial(6,p) but it is Binomial(2,p) (2 trials in 6 seconds).
- The time interval Δ of each Bernoulli is called a frame.
  - Number of trials equals to t/Δ

Arrival/Success rate λ

- If p is the success rate at Δ units of time, then λ is the success rate per 1 unit of time
  - $\lambda = p/\Delta$
- T is the time between two success. (Y was the number of trials (Δs) between two success.
  - $T = Y\Delta$

Transition probabilities of a Binomial Process

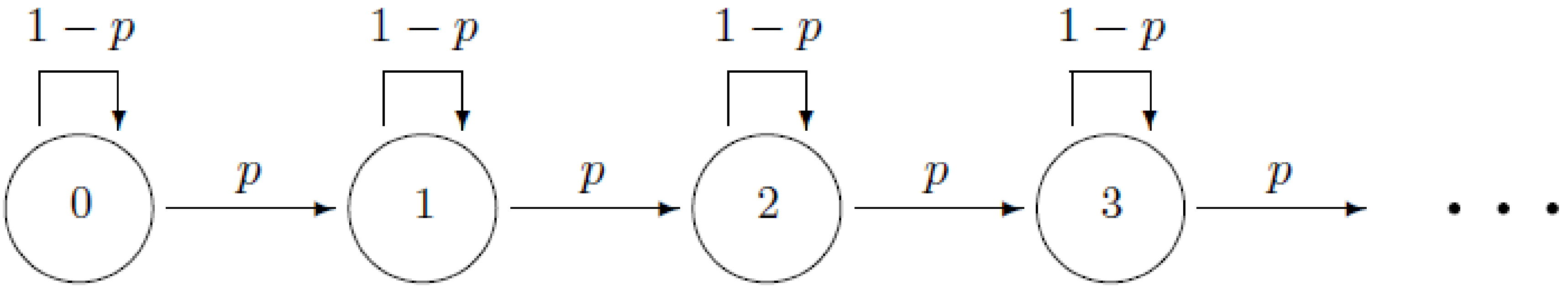

FIGURE 6.8: *Transition diagram for a Binomial counting process.*

- Is it regular?

Poisson counting process

- When the frame Δ approaches 0, we approach a continuous-time counting process.
  - Note that as Δ approaches 0, p also approaches 0.
- Taking the success/arrival rate per unit of time, λ, as constant, we may model such continuous counting processes.
- $X(t)$ becomes a Poisson(λt) variable. T becomes an Exponential(λ) variable.

Using the Gamma-Poisson formula

- Recall that Poisson problems could be solved using the Gamma distribution (Chapter 4)
  - Gamma-Poisson formula (Eq. 4.14)
- Time needed for the *k*th success, T_k, is a Gamma(k, λ) variable
- $P(T_k \leq t) = P(k \text{ successes before time } t) = P(X(t) \geq k)$
  - T_k is Gamma(k, λ) and $X(t)$ is Poisson(λt)

Poisson Process

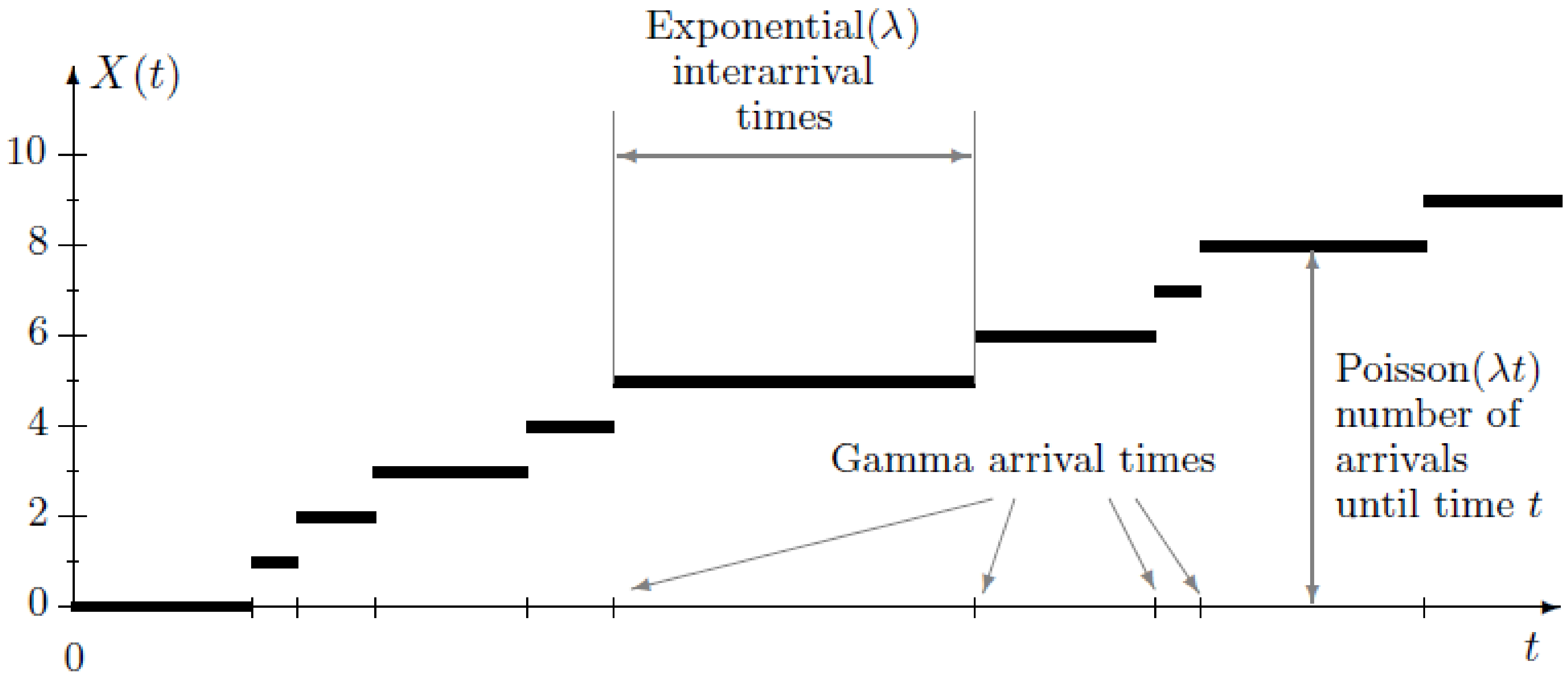

FIGURE 6.10: *Poisson process (sample path).*

Simulation of Stochastic Processes

- We can use random sampling techniques we learned in Chapter 5 to simulate stochastic processes.
  - For example: state transitions are discrete with specific *pmf*s, which could be simulated by using Algorithm 5.1 (or the Alias method for efficiency)
- Steady-state distributions of a regular Markov chain can also be found using an iterative simulation and checking whether two successive state-distributions are equal (or very close).