

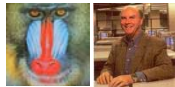
Multiple Alignment

Outline

- Problem definition
- Can we use Dynamic Programming to solve MSA?
- Progressive Alignment
- ClustalW
- Scoring Multiple Alignments
 - Entropy
 - Sum of Pairs (SP) Score

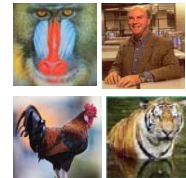
Multiple Alignment versus Pairwise Alignment

- Up until now we have only tried to align two sequences.



Multiple Alignment versus Pairwise Alignment

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- What about more than two? And what for?



Multiple Alignment versus Pairwise Alignment

- Up until now we have only tried to align two sequences.
- What about more than two? And what for?
- A faint similarity between two sequences becomes significant if present in many
- Multiple alignments can reveal subtle similarities that pairwise alignments do not reveal



Multiple alignment

- One of the most essential tools in molecular biology
 - Finding highly conserved subregions or embedded patterns of a set of biological sequences
 - Conserved regions usually are key functional regions, prime targets for drug developments
 - Estimation of evolutionary distance between sequences
 - Prediction of protein secondary/tertiary structure
- Practically useful methods only since 1987 (D. Sankoff)
 - Before 1987 they were constructed by hand
 - Dynamic programming is expensive

Multiple Sequence Alignment (MSA)

- What is multiple sequence alignment?
- Given k sequences:

```
VTISCTGSSSNIGAGNHVKWYQQLPG
VTISCTGTSSNIGS--ITVNWYQQLPG
LRLSCSSSGFIFSS--YAMYVVRQAPG
LSLTCTVSGTSFDD--YYSTWVRQPPG
PEVTCVVVDVSHEDPQVKFNWYVDG--
ATLVCLISDFYPGA--VTVAWKADS--
AALGCLVKDYFPEP--VTVSWNSG--
VSLTCLVKGFPSPD--IAVEWESNG--
```

Multiple Sequence Alignment (MSA)

- An MSA of these sequences:

```
VTISCTGSSSNIGAG-NHVKWYQQLPG
VTISCTGTSSNIGS--ITVNWYQQLPG
LRLSCSSSGFIFSS--YAMYVVRQAPG
LSLTCTVSGTSFDD--YYSTWVRQPPG
PEVTCVVVDVSHEDPQVKFNWYVDG--
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```

Conserved residues

Multiple Sequence Alignment (MSA)

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```

Conserved regions

Multiple Sequence Alignment (MSA)

- An MSA of these sequences:

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```

Patterns? Positions 1 and 3 are hydrophobic residues

Multiple Sequence Alignment (MSA)

- An MSA of these sequences:

```
VTISCTGSSSNIGAG-NHVKWYQQLPG
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```

Conserved residues, regions, patterns

MSA Warnings

- MSA algorithms work under the assumption that they are aligning related sequences
- They will align ANYTHING they are given, even if unrelated
- If it just “looks wrong” it probably is

Generalizing the Notion of Pairwise Alignment

- Alignment of 2 sequences is represented as a 2-row matrix
- In a similar way, we represent alignment of 3 sequences as a 3-row matrix

```
A T _ G C G _
A _ C G T _ A
A T C A C _ A
```

- Score: more conserved columns, better alignment

Alignments = Paths in k dimensional grids

- Align 3 sequences: ATGC, AATC, ATGC

	A	--	T	G	C
--	---	----	---	---	---

	A	A	T	--	C
--	---	---	---	----	---

	--	A	T	G	C
--	----	---	---	---	---

Alignment Paths

0	1	1	2	3	4	
	A	--	T	G	C	x coordinate

	A	A	T	--	C
--	---	---	---	----	---

	--	A	T	G	C
--	----	---	---	---	---

Alignment Paths

0	1	1	2	3	4	x coordinate
	A	--	T	G	C	

0	1	2	3	3	4	y coordinate
	A	A	T	--	C	

	--	A	T	G	C
--	----	---	---	---	---

-

Alignment Paths

0	1	1	2	3	4	x coordinate
	A	--	T	G	C	

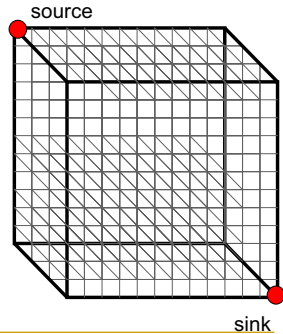
0	1	2	3	3	4	y coordinate
	A	A	T	--	C	

0	0	1	2	3	4	z coordinate
	--	A	T	G	C	

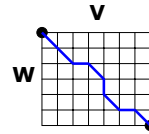
- Resulting path in (x,y,z) space:
 $(0,0,0) \rightarrow (1,1,0) \rightarrow (1,2,1) \rightarrow (2,3,2) \rightarrow (3,3,3) \rightarrow (4,4,4)$

Aligning Three Sequences

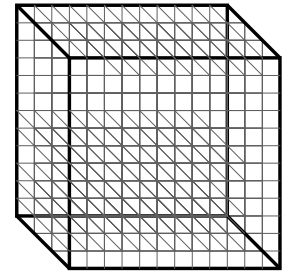
- Same strategy as aligning two sequences
- Use a 3-D matrix, with each axis representing a sequence to align
- For global alignments, go from source to sink



2-D vs 3-D Alignment Grid

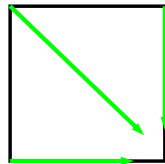
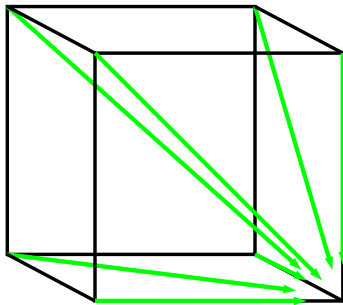


2-D alignment matrix



3-D alignment matrix

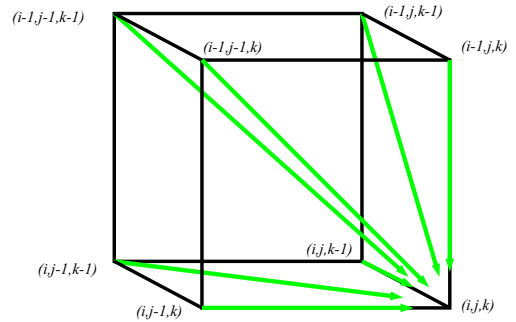
2-D cell versus 3-D Alignment Cell



In **2-D**, 3 edges
in each unit
square

In **3-D**, 7 edges
in each unit
cube

Architecture of 3-D Alignment Cell



Multiple Alignment: Dynamic Programming

$$s_{i,j,k} = \max \begin{cases} s_{i-1,j-1,k-1} + \delta(v_p, w_p, u_k) & \text{cube diagonal:} \\ s_{i-1,j-1,k} + \delta(v_p, w_p, _) & \text{no indels} \\ s_{i-1,j,k-1} + \delta(v_p, _, u_k) & \text{face diagonal:} \\ s_{i,j-1,k-1} + \delta(_, w_p, u_k) & \text{one indel} \\ s_{i-1,j,k} + \delta(v_p, _, _) & \text{edge diagonal:} \\ s_{i,j-1,k} + \delta(_, w_p, _) & \text{two indels} \\ s_{i,j,k-1} + \delta(_, _, u_k) & \end{cases}$$

- $\delta(x, y, z)$ is an entry in the 3-D scoring matrix

Multiple Alignment: Running Time

- For 3 sequences of length n , the run time is $7n^3$; $O(n^3)$
- For k sequences, build a k -dimensional matrix, with run time $(2^k-1)(n^k)$; $O(2^k n^k)$
- Conclusion: dynamic programming approach for alignment between two sequences is easily extended to k sequences but it is impractical due to exponential running time

Multiple Alignment Induces Pairwise Alignments

Every multiple alignment induces pairwise alignments

```
x: AC-GCGG-C
y: AC-GC-GAG
z: GCCGC-GAG
```

Induces:

```
x: ACGCGG-C; x: AC-GCGG-C; y: AC-GCGAG
y: ACGC-GAC; z: GCCGC-GAG; z: GCCGCAGAG
```

Reverse Problem: Constructing Multiple Alignment from Pairwise Alignments

Given 3 **arbitrary** pairwise alignments:

```
x: ACGCTGG-C; x: AC-GCTGG-C; y: AC-GC-GAG
y: ACGC--GAC; z: GCCGCA-GAG; z: GCCGCAGAG
```

can we construct a multiple alignment that induces them?

Reverse Problem: Constructing Multiple Alignment from Pairwise Alignments

Given 3 **arbitrary** pairwise alignments:

```
x: ACGCTGG-C; x: AC-GCTGG-C; y: AC-GC-GAG
y: ACGC--GAC; z: GCCGCA-GAG; z: GCCGCAGAG
```

can we construct a multiple alignment that induces them?

NOT ALWAYS

Pairwise alignments may be inconsistent

Inferring Multiple Alignment from Pairwise Alignments

- From an optimal multiple alignment, we can infer pairwise alignments between all pairs of sequences, but they are not necessarily optimal
- It is difficult to infer a "good" multiple alignment from optimal pairwise alignments between all sequences

Combining Optimal Pairwise Alignments into Multiple Alignment

Can combine pairwise alignments into multiple alignment



Can **not** combine pairwise alignments into multiple alignment



Consensus String of a Multiple Alignment

```
- A G G C T A T C A C C T G
T A G - C T A C C A - - G
C A G - C T A C C A - - G
C A G - C T A T C A C - G G
C A G - C T A T C G C - G G
```

Consensus String: C A G C T A T C A C G G

- The *consensus string* S_M derived from multiple alignment M is the concatenation of the consensus characters for each column of M .
 - The *consensus character* for column i is the character that minimizes the summed distance to it from all the characters in column i . (i.e., if match and mismatch scores are equal for all symbols, the majority symbol is the consensus character)

Profile Representation of Multiple Alignment

```

- A G G C T A T C A C C T G
T A G - C T A C C A - - G
C A G - C T A C C A - - G
C A G - C T A T C A C - G G
C A G - C T A T C G C - G G

A   1           1       .8
C   .6         1       .4 1   .6 .2
G   .2         1.2     .2   .4 1
T   .2         1       .6       .2
-   .2         .8           .4 .8 .4
    
```

Profile Representation of Multiple Alignment

```

- A G G C T A T C A C C T G
T A G - C T A C C A - - G
C A G - C T A C C A - - G
C A G - C T A T C A C - G G
C A G - C T A T C G C - G G

A   1           1       .8
C   .6         1       .4 1   .6 .2
G   .2         1.2     .2   .4 1
T   .2         1       .6       .2
-   .2         .8           .4 .8 .4
    
```

Earlier, we were aligning a **sequence against a sequence**

Can we align a **sequence against a profile?**

Can we align a **profile against a profile?**

Aligning alignments

- Given two alignments, can we align them?

```

x GGGCACTGCAT
y GGTACGTC--
z GGGAACTGCAG

w GGACGTACC--
v GGACCT-----

Alignment 1
Alignment 2
    
```

Aligning alignments

- Given two alignments, can we align them?
- Hint: use alignment of corresponding profiles

```

x GGGCACTGCAT
y GGTACGTC--
z GGGAACTGCAG
w GGACGTACC--
v GGACCT-----

Combined Alignment
    
```

Multiple Alignment: Greedy Approach

- Choose most similar pair of strings and combine into a profile, thereby reducing alignment of k sequences to an alignment of $k-1$ sequences/profiles. **Repeat**
- This is a heuristic greedy method

$$\left\{ \begin{array}{l} u_1 = \text{ACGTACGTACGT} \dots \\ u_2 = \text{TTAATTAATTAA} \dots \\ u_3 = \text{ACTACTACTACT} \dots \\ \dots \\ u_k = \text{CCGGCCGGCCGG} \end{array} \right\} \xrightarrow{\quad} \left\{ \begin{array}{l} u_1 = \text{ACg/tTACg/tTACg/cT} \dots \\ u_2 = \text{TTAATTAATTAA} \dots \\ \dots \\ u_k = \text{CCGGCCGGCCGG} \dots \end{array} \right\} \quad k-1$$

Greedy Approach: Example

- Consider these 4 sequences

```

s1 GATTCA
s2 GTCTGA
s3 GATATT
s4 GTCAGC
    
```

Greedy Approach: Example (cont'd)

- There are $\binom{4}{2} = 6$ possible alignments

s_2 GTCTGA	s_1 GATTCA--
s_4 GTCAGC (score = 2)	s_4 G-T-CAGC (score = 0)
s_1 GAT-TCA	s_2 G-TCTGA
s_2 G-TCTGA (score = 1)	s_3 GATAT-T (score = -1)
s_1 GAT-TCA	s_3 GAT-ATT
s_3 GATAT-T (score = 1)	s_4 G-T-CAGC (score = -1)

Greedy Approach: Example (cont'd)

s_2 and s_4 are closest; combine:

s_2 GTCTGA	} $s_{2,4}$ GTCT/aGa/cA
s_4 GTCAGC	

(profile)

new set of 3 sequences:

s_1	GATTCA
s_3	GATATT
$s_{2,4}$	GTCT/aGa/c

Progressive Alignment

- Progressive alignment* is a variation of greedy algorithm with a somewhat more intelligent strategy for choosing the order of alignments.
- Progressive alignment works well for close sequences, but deteriorates for distant sequences
 - Gaps in consensus string are permanent
 - Use profiles to compare sequences

Star alignment

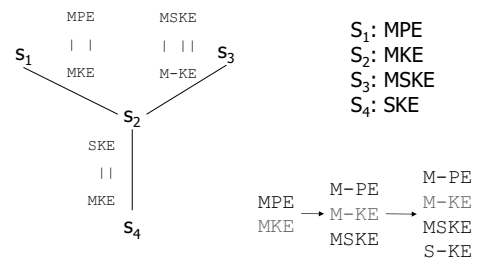
- Heuristic method for multiple sequence alignments
- Select a sequence c as the center of the star
- For each sequence x_1, \dots, x_k such that index $i \neq c$, perform a Needleman-Wunsch global alignment
- Aggregate alignments with the principle "once a gap, always a gap."

Choosing a center

- Try them all and pick the one which is most similar to all of the sequences
- Let $S(x_i, x_j)$ be the optimal score between sequences x_i and x_j .
- Calculate all $O(k^2)$ alignments, and choose as x_c the sequence x_i that maximizes the following

$$\sum_{j \neq i} S(x_i, x_j)$$

Star alignment example



ClustalW: another example

S_1 ALSK
 S_2 TNSD
 S_3 NASK
 S_4 NTSD

ClustalW example

S_1 ALSK
 S_2 TNSD
 S_3 NASK
 S_4 NTSD

All pairwise alignments

	S_1	S_2	S_3	S_4
S_1	0	9	4	7
S_2		0	8	3
S_3			0	7
S_4				0

Distance Matrix

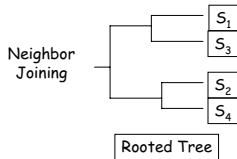
ClustalW example

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	S_1	S_2	S_3	S_4
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S_3			0	7
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Distance Matrix



ClustalW example

S_1 ALSK
 S_2 TNSD
 S_3 NASK
 S_4 NTSD

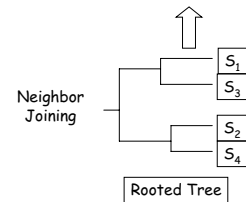
Multiple Alignment Steps

1. Align S_1 with S_3
2. Align S_2 with S_4
3. Align (S_1, S_3) with (S_2, S_4)

All pairwise alignments

	S_1	S_2	S_3	S_4
S_1	0	9	4	7
S_2		0	8	3
S_3			0	7
S_4				0

Distance Matrix



ClustalW example

S_1 ALSK
 S_2 TNSD
 S_3 NASK
 S_4 NTSD

Multiple Alignment

-ALSK
 NA-SK
 -TNSD
 NA-SK
 -TNSD
 NT-SD

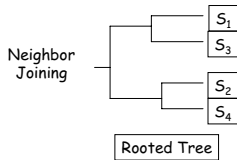
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All pairwise alignments

	S_1	S_2	S_3	S_4
S_1	0	9	4	7
S_2		0	8	3
S_3			0	7
S_4				0

Distance Matrix



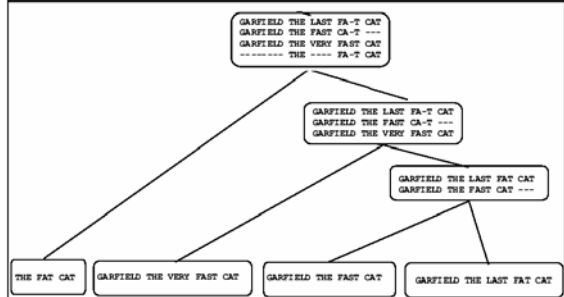
Other progressive approaches

- PILEUP
 - Similar to CLUSTALW
 - Uses UPGMA to produce tree

Problems with progressive alignments

- Depend on pairwise alignments
- If sequences are very distantly related, much higher likelihood of errors
- Care must be made in choosing scoring matrices and penalties

Figure 1. Limits of the progressive strategy.



This example shows how a progressive alignment strategy can be misled. In the initial alignment of sequences 1 and 2, ClustalW has a choice between aligning CAT with CAT and making an internal gap or making a mismatch between C and F and having a terminal gap. Since terminal gaps are much cheaper than internal, the ClustalW scoring scheme prefers the former. In the next stage, when the extra sequence is added, it turns out that properly aligning the two CATs in the previous stage would have led to a better scoring sum-of-pairs multiple alignment.

Iterative refinement in progressive alignment

Another problem of progressive alignment:

- Initial alignments are “frozen” even when new evidence comes

Example:

```

x:  GAAGTT
y:  GAC-TT  >  Frozen!

z:  GAACTG
w:  GTACTG  >  Now clear that correct y = GA-C TT
  
```

Evaluating multiple alignments

- Balibase benchmark (Thompson, 1999)
- De-facto standard for assessing the quality of a multiple alignment tool
- Manually refined multiple sequence alignments
- Quality measured by how good it matches the core blocks
- Another benchmark: SABmark benchmark
 - Based on protein structural families

Scoring multiple alignments

- Ideally, a scoring scheme should
 - Penalize variations in conserved positions higher
 - Relate sequences by a phylogenetic tree
 - Tree alignment
- Usually assume
 - Independence of columns
 - Quality computation
 - Entropy-based scoring
 - Compute the Shannon entropy of each column
 - Sum-of-pairs (SP) score

Multiple Alignments: Scoring

- Number of matches (multiple longest common subsequence score)
- Entropy score
- Sum of pairs (SP-Score)

Multiple LCS Score

- A column is a “match” if all the letters in the column are the same

AAA
AAA
AAT
ATC

- Only good for very similar sequences

Entropy

- Define frequencies for the occurrence of each letter in each column of multiple alignment
 - $p_A = 1, p_T = p_G = p_C = 0$ (1st column)
 - $p_A = 0.75, p_T = 0.25, p_G = p_C = 0$ (2nd column)
 - $p_A = 0.50, p_T = 0.25, p_C = 0.25, p_G = 0$ (3rd column)
- Compute entropy of each column

$$-\sum_{x=A,T,G,C} p_x \log p_x$$

AAA
AAA
AAT
ATC

Entropy: Example

$$\text{entropy} \begin{pmatrix} A \\ A \\ A \\ A \end{pmatrix} = 0 \quad \text{Best case}$$

$$\text{Worst case} \quad \text{entropy} \begin{pmatrix} A \\ T \\ G \\ C \end{pmatrix} = -\sum \frac{1}{4} \log \frac{1}{4} = -4 \left(\frac{1}{4} * -2 \right) = 2$$

Multiple Alignment: Entropy Score

Entropy for a multiple alignment is the sum of entropies of its columns:

$$\sum_{\text{over all columns}} -\sum_{x=A,T,G,C} p_x \log p_x$$

Entropy of an Alignment: Example

$$-(p_A \log p_A + p_C \log p_C + p_G \log p_G + p_T \log p_T)$$

A	A	A
A	C	C
A	C	G
A	C	T

•Column 1 = $-[1 * \log(1) + 0 * \log 0 + 0 * \log 0 + 0 * \log 0]$
= 0

•Column 2 = $-[(1/4) * \log(1/4) + (3/4) * \log(3/4) + 0 * \log 0 + 0 * \log 0]$
= $-[(1/4) * (-2) + (3/4) * (-1.15)] = +0.811$

•Column 3 = $-[(1/4) * \log(1/4) + (1/4) * \log(1/4) + (1/4) * \log(1/4) + (1/4) * \log(1/4)]$
= $4 * -(1/4) * (-2) = +2.0$

•Alignment Entropy = $0 + 0.811 + 2.0 = +2.811$

Multiple Alignment Induces Pairwise Alignments

Every multiple alignment induces pairwise alignments

x: AC-GCGG-C
y: AC-GC-GAG
z: GCCGC-GAG

Induces:

x: ACGCGG-C; x: AC-GCGG-C; y: AC-GCGAG
y: ACGC-GAC; z: GCCGC-GAG; z: GCCGCGAG

Sum of Pairs (SP) Scoring

- SP scoring is the standard method for scoring multiple sequence alignments.
- Columns are scored by a 'sum of pairs' function using a substitution matrix (PAM or BLOSUM)
- Assumes statistical independence for the columns, does not use a phylogenetic tree.

Sum of Pairs Score (SP-Score)

- Consider pairwise alignment of sequences a_i and a_j imposed by a multiple alignment of k sequences
- Denote the score of this suboptimal (not necessarily optimal) pairwise alignment as $s^*(a_i, a_j)$
- Sum up the pairwise scores for a multiple alignment:

$$s(a_1, \dots, a_k) = \sum_{i,j} s^*(a_i, a_j)$$

Computing SP-Score

Aligning 4 sequences: 6 pairwise alignments

Given a_1, a_2, a_3, a_4 :

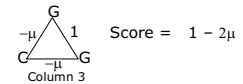
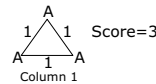
$$s(a_1 \dots a_4) = \sum s^*(a_i, a_j) = s^*(a_1, a_2) + s^*(a_1, a_3) + s^*(a_1, a_4) + s^*(a_2, a_3) + s^*(a_2, a_4) + s^*(a_3, a_4)$$

SP-Score: Example

a_1 ATG-C-AAT
 a_2 A-G-CATAT
 a_3 ATCCCATTT

$$S(a_1 \dots a_k) = \sum_{i,j} S^*(a_i, a_j) \leftarrow \binom{n}{2} \text{ Pairs of Sequences}$$

May also calculate the scores column by column:



Example

- Compute Sum of Pairs Score of the following multiple alignment with match = 3, mismatch = -1, $S(X,-) = -1$, $S(-,-) = 0$

X: G T A C G
 Y: T G C C G
 Z: C G G C C
 W: C G G A C
 -2 6 -2 6 2

Sum of pairs = -2+6-2+6+2 = 10

Multiple alignment tools

- Clustal W (Thompson, 1994)
 - Most popular
- PRRP (Gotoh, 1993)
- HMMT (Eddy, 1995)
- DIALIGN (Morgenstern, 1998)
- T-Coffee (Notredame, 2000)
- MUSCLE (Edgar, 2004)
- Align-m (Walle, 2004)
- PROBCONS (Do, 2004)

Table 1. Some recent and less recent available methods for MSAs.

Name	Algorithm	URL
MSA	Exact	http://www.ibc.wustl.edu/ibc/rsa.html
DCA	Exact (requires MSA)	http://bibiserv.techfak.uni-bielefeld.de/dca
OMA	Iterative DCA	http://bibiserv.techfak.uni-bielefeld.de/oma
ClustalW, ClustalX	Progressive	ftp://ftp-igbmc.u-strasbg.fr/pub/dustalW or dustalX
MultAlin	Progressive	http://www.toulouse.inra.fr/multalin.html
Dialign	Consistency-based	http://www.gsf.de/biodiv/dialign.html
Com-Align	Consistency-based	http://www.dairi.au.dk/~ocaprani
T-Coffee	Consistency-based/progressive	http://igs-server.cnrs-mrs.fr/~cnotred
Praline	Iterative/progressive	jheing@nimr.mrc.ac.uk
IterAlign	Iterative	http://giotto.Stanford.edu/~luciano/iteralign.html
Prnp	Iterative/Stochastic	ftp://ftp.genome.ad.jp/pub/genome/saitama-cc/
SAM	Iterative/Stochastic/HMM	rph@ce.ucsc.edu
HMMER	Iterative/Stochastic/HMM	http://hmmr.wustl.edu/
SAGA	Iterative/Stochastic/GA	http://igs-server.cnrs-mrs.fr/~cnotred
GA	Iterative/Stochastic/GA	c.zhang@watnow.uwaterloo.ca

from: C. Notredame, "Recent progresses in multiple alignment: a survey", *Pharmacogenomics* (2002) 3(1)

Useful links

<http://cnx.org/content/m11036/latest/>

<http://www.biokemi.uu.se/Utbildning/Exercises/ClustalX/index.shtm>

http://bioinformatics.weizmann.ac.il/~pietro/Making_and_using_protein_MA/

http://homepage.usask.ca/~ct1271/857/paper1_overview.shtml

<http://journal-ci.csse.monash.edu.au/ci/vol04/mulali/mulali.html>